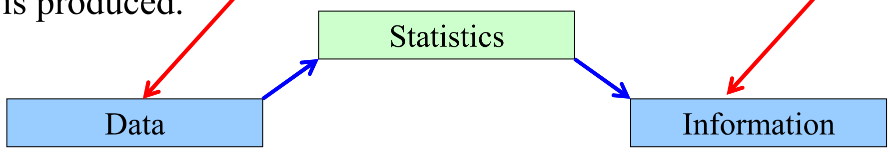
Chapter Two

Graphical and Tabular Descriptive Techniques

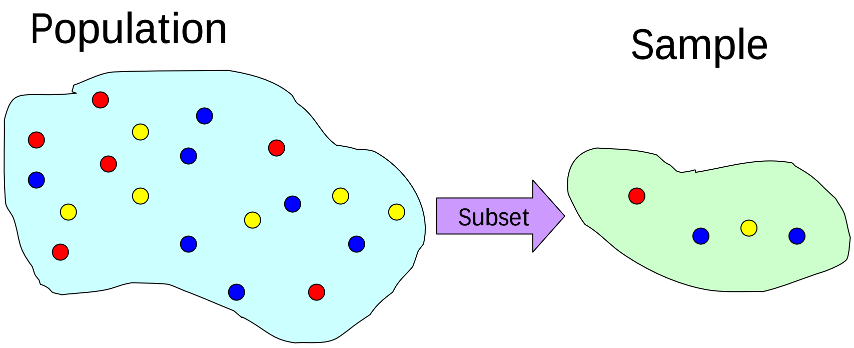
Introduction & Re-cap...

Descriptive statistics involves arranging, summarizing, and presenting a *set of data* in such a way that useful *information* is produced.



Its methods make use of graphical techniques and numerical descriptive measures (such as averages) to summarize and present the data.

Populations & Samples



The graphical & tabular methods presented here apply to both entire populations ***and*** samples drawn from populations.

Definitions…

A **variable** is some characteristic of a population or sample.

E.g. student grades.

Typically denoted with a capital letter: X, Y, Z…

The **values** of the variable are the range of possible values for a variable.

E.g. student marks (0..100)

Data are the ***observed values*** of a variable.

E.g. student marks: {67, 74, 71, 83, 93, 55, 48}

Types of Data & Information

Data (at least for purposes of Statistics) fall into three main groups:

Interval Data

Nominal Data

Ordinal Data

Interval Data...

Interval data

- Real numbers, i.e. heights, weights, prices, etc.
- Also referred to as **quantitative** or **numerical**.

Arithmetic operations can be performed on Interval Data, thus its meaningful to talk about 2*Height, or Price + $1, and so on.

Nominal Data...

Nominal Data

• The values of **nominal** data are ***categories.***

E.g. responses to questions about marital status, coded as:

Single = 1, Married = 2, Divorced = 3, Widowed = 4

These data are **categorical** in nature; arithmetic operations don't make any sense (e.g. does Widowed ÷ 2 = Married?!)

Nominal data are also called **qualitative** or **categorical**.

Ordinal Data...

Ordinal Data appear to be categorical in nature, but their values have an ***order***; a ranking to them:

E.g. College course rating system:

poor = 1, fair = 2, good = 3, very good = 4, excellent = 5

While its still not meaningful to do arithmetic on this data (e.g. does 2*fair = very good?!), we can say things like:

`excellent > poor` or **`fair < very good`**

That is, order is maintained no matter what numeric values are assigned to each category.

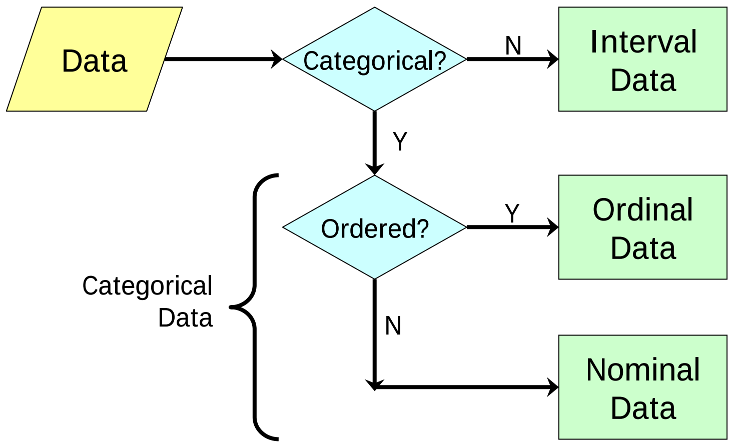
Calculations for Types of Data

As mentioned above,

- All calculations are permitted on **interval** data.
- Only calculations involving a ranking process are allowed for **ordinal** data.
- No calculations are allowed for **nominal** data, save counting the number of observations in each category.

This lends itself to the following "hierarchy of data"…

Types of Data & Information…



Hierarchy of Data...

Interval

Values are real numbers.
All calculations are valid.
Data may be treated as ordinal or nominal.

Ordinal

Values must represent the ranked order of the data.
Calculations based on an ordering process are valid.
Data may be treated as nominal but not as interval.

Nominal

Values are the arbitrary numbers that represent categories.
Only calculations based on the frequencies of occurrence are valid.
Data may not be treated as ordinal or interval.

Graphical & Tabular Techniques for Nominal Data...

The only allowable calculation on nominal data is to count the frequency of each value of the variable.

We can summarize the data in a table that presents the categories and their counts called a ***frequency distribution.***

A ***relative frequency distribution*** lists the categories and the proportion with which each occurs.

Example 2.1 Light Beer Preference Survey

In 2006 total light beer sales in the United States was approximately 3 million gallons

With this large a market breweries often need to know more about who is buying their product.

The marketing manager of a major brewery wanted to analyze the light beer sales among college and university students who do drink light beer.

A random sample of 285 graduating students was asked to report which of the following is their favorite light beer.

Example 2.1

1. Budweiser Light
2. Busch Light
3. Coors Light
4. Michelob Light
5. Miller Lite
6. Natural Light
7. Other brand

The responses were recorded using the codes. Construct a frequency and relative frequency distribution for these data and graphically summarize the data by producing a bar chart and a pie chart.

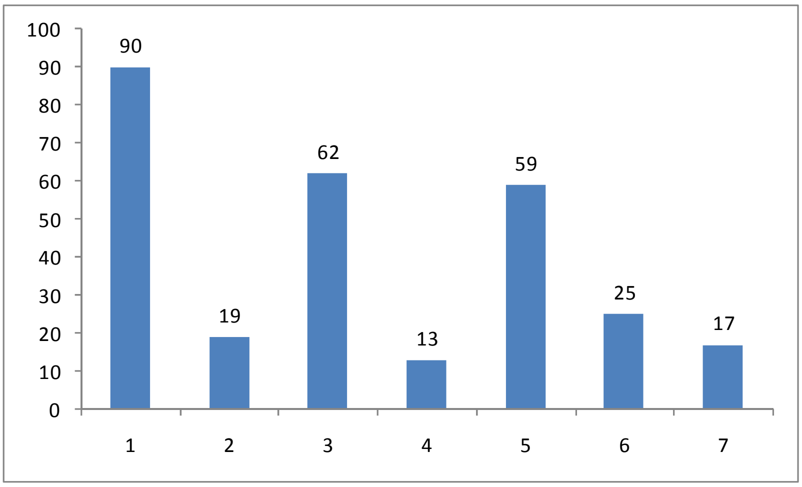
Example 2.1 Xm02-01*

1	1	1	1	2	4	3	5	1	3	1	3	7	5	1
1	5	2	1	5	1	3	3	3	1	1	5	3	1	5
5	1	1	3	3	5	5	6	3	5	3	5	5	5	1
1	2	1	1	5	5	3	2	1	6	1	1	4	5	1
3	3	5	4	7	6	6	4	4	6	5	2	1	1	5
3	3	1	3	5	3	3	7	3	7	2	1	5	7	
3	6	2	6	3	6	6	6	5	6	1	1	6	3	
7	1	1	1	5	1	3	1	3	7	7	2	1	1	
2	5	3	1	1	3	1	1	7	5	3	2	1	1	
6	5	7	1	3	2	1	3	1	1	7	5	5	6	
1	4	6	1	3	1	1	5	5	5	5	1	5	5	
6	1	3	3	1	3	7	1	1	1	2	4	1	1	
3	3	7	5	5	1	1	3	5	1	5	4	5	3	
4	1	4	5	3	1	5	3	3	3	1	1	5	3	
5	6	4	3	5	6	4	6	5	5	5	5	3	1	
2	3	2	7	5	1	6	6	2	3	3	3	1	1	
5	1	4	6	3	5	1	1	2	1	5	6	1	1	
5	1	3	5	1	1	1	3	7	3	1	6	3	1	
2	2	5	1	3	5	5	2	3	1	1	3	6	1	
1	1	1	7	3	1	5	3	3	3	5	3	1	7	

Frequency and Relative Frequency Distributions

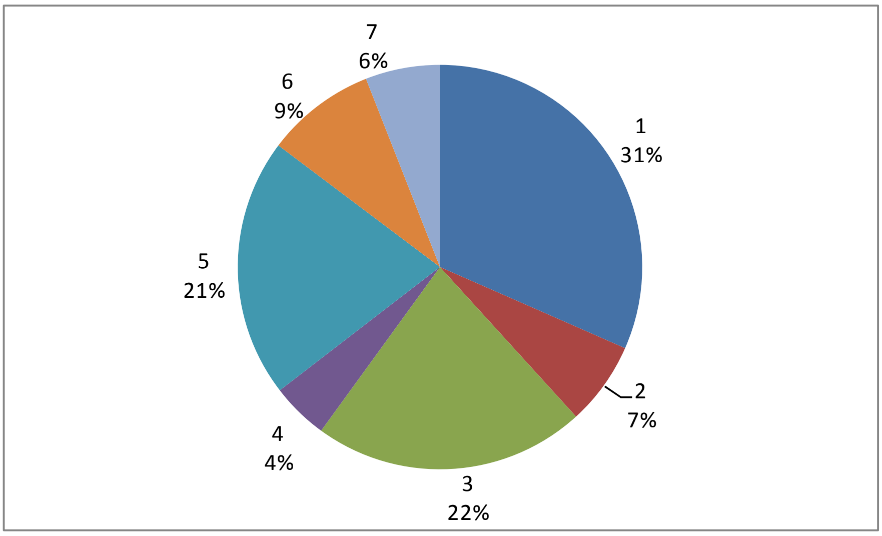
Light Beer Brand	Frequency	Relative Frequency
Budweiser Light	90	31.6%
Busch Light	19	6.7
Coors Light	62	21.8
Michelob Light	13	4.6
Miller Lite	59	20.7
Natural Light	25	8.8
Other brands	17	6.0
Total	285	100

Nominal Data (Frequency)



Bar Charts are often used to display ***frequencies…***

Nominal Data (Relative Frequency)

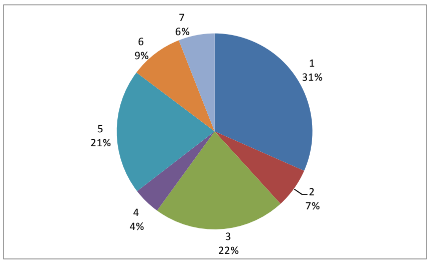


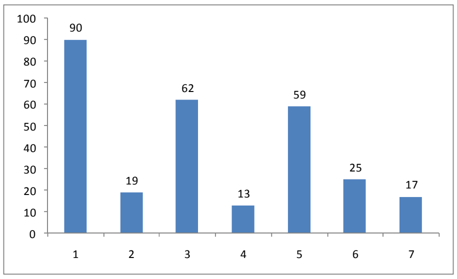
Pie Charts show ***relative frequencies…***

Nominal Data

Light Beer Brand	Frequency	Relative Frequency
Budweiser Light	90	31.6%
Busch Light	19	6.7
Coors Light	62	21.8
Michelob Light	13	4.6
Miller Lite	59	20.7
Natural Light	25	8.8
Other brands	17	6.0

It all the same ***information,***
(based on the same ***data***).
Just different ***presentation.***





Example 2.2

Table 2.3 lists the total energy consumption of the United States from all sources in 2005.

To make it easier to see the details the table measures the heat content in metric tons (1,000 kilograms) of oil equivalent.

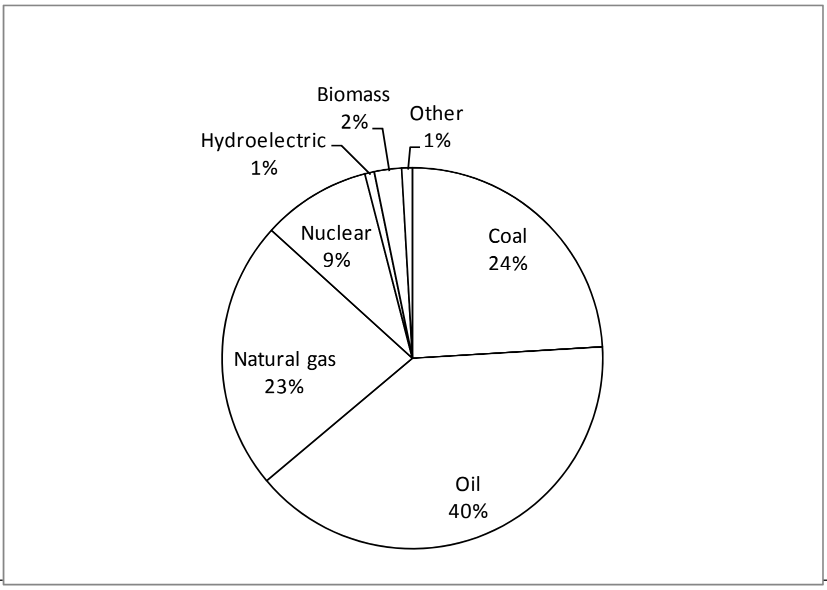
For example, the United States burned an amount of coal and coal products equivalent to 545,259 metric tons of oil.

Use an appropriate graphical technique to depict these figures.

Table 2.3 Xm02-02*

Non-Renewable Energy Sources	Heat Content
Coal & coal products	545,258
Oil	903,440
Natural Gas	517,881
Nuclear	209,890
Renewable Energy Sources	
Hydroelectric	18,251
Solid Biomass	52,473
Other (Liquid biomass, geothermal, solar, wind, and tide, wave, & Ocean)	20,533
Total	2,267,726

Example 2.2



Graphical Techniques for Interval Data

There are several graphical methods that are used when the data are ***interval*** (i.e. numeric, non-categorical).

The most important of these graphical methods is the ***histogram***.

The histogram is not only a powerful graphical technique used to ***summarize*** interval data, but it is also used to help ***explain*** probabilities.

Example 2.4

Following deregulation of telephone service, several new companies were created to compete in the business of providing long-distance telephone service. In almost all cases these companies competed on price since the service each offered is similar. Pricing a service or product in the face of stiff competition is very difficult. Factors to be considered include supply, demand, price elasticity, and the actions of competitors. Long-distance packages may employ per-minute charges, a flat monthly rate, or some combination of the two. Determining the appropriate rate structure is facilitated by acquiring information about the behaviors of customers and in particular the size of monthly long-distance bills.

Example 2.4

As part of a larger study, a long-distance company wanted to acquire information about the monthly bills of new subscribers in the first month after signing with the company. The company's marketing manager conducted a survey of 200 new residential subscribers wherein the first month's bills were recorded. These data are stored in file Xm02-04. The general manager planned to present his findings to senior executives. What information can be extracted from these data?

Example 2.4

In Example 2.1 we created a frequency distribution of the 5 categories. In this example we also create a frequency distribution by counting the number of observations that fall into a series of intervals, called classes.

I'll explain later why I chose the classes I use below.

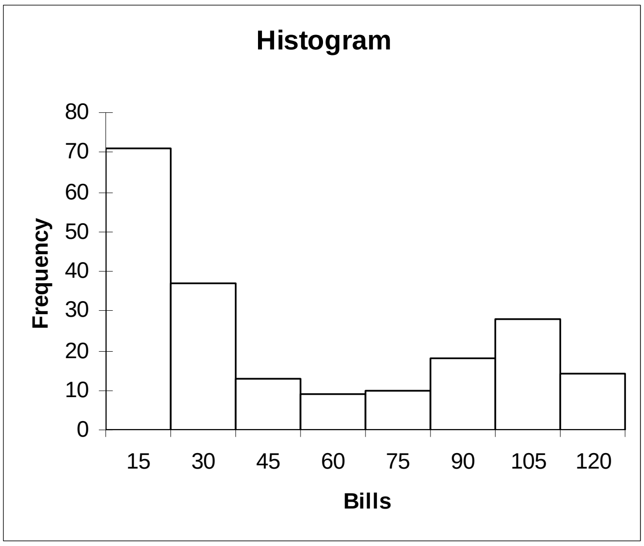
Example 2.4

We have chosen eight classes defined in such a way that each observation falls into one and only one class. These classes are defined as follows:

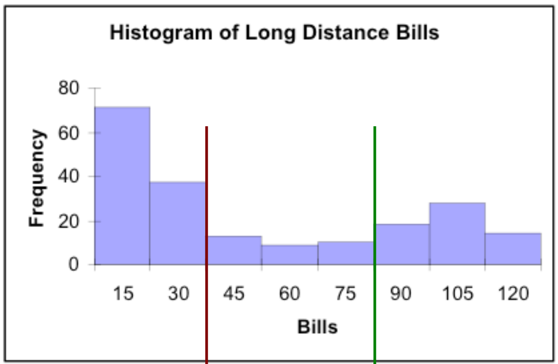
Classes

Amounts that are less than or equal to 15
Amounts that are more than 15 but less than or equal to 30
Amounts that are more than 30 but less than or equal to 45
Amounts that are more than 45 but less than or equal to 60
Amounts that are more than 60 but less than or equal to 75
Amounts that are more than 75 but less than or equal to 90
Amounts that are more than 90 but less than or equal to 105
Amounts that are more than 105 but less than or equal to 120

Example 2.4



Interpret...



about half (71+37=108) of the bills are "small", i.e. less than $30

(18+28+14=60)÷200 = 30% i.e. nearly a third of the phone bills are $90 or more.

There are only a few telephone bills in the middle range.

Building a Histogram...

1) Collect the Data
2) Create a frequency distribution for the data…

   How?

   a) Determine the number of ***classes*** to use…

      How?

Refer to table 2.6:

Table 2.6 Approximate Number of Classes in Frequency Distributions

Number of Observations	Number of Classes
Less than 50	5 - 7
50 - 200	7 - 9
200 - 500	9 - 10
500 - 1,000	10 - 11
1,000 - 5,000	11 - 13
5,000 - 50,000	13 - 17
More than 50,000	17 - 20

With 200 observations, we should have between 7 & 10 classes...

Alternative, we could use Sturges' formula:
Number of class intervals = **1 + 3.3 log (n)**

Building a Histogram...

1) Collect the Data
2) Create a frequency distribution for the data…
   How?
   a) Determine the number of ***classes*** to use. [8]
   b) Determine how large to make each class…
      How?
      Look at the ***range*** of the data, that is,
      Range = Largest Observation – Smallest Observation
      Range = \$119.63 – \$0 = \$119.63
      Then each class width becomes:
      Range ÷ (# classes) = 119.63 ÷ 8 ≈ 15

Building a Histogram...

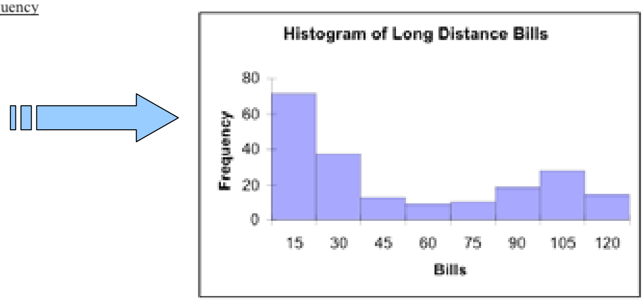
Table 2.5 Frequency Distribution of the Long-Distance Bills in Example 2.4

Class Limits	Frequency
0 to 15	71
15 to 30	37
30 to 45	13
45 to 60	9
60 to 75	10
75 to 90	18
90 to 105	28
105 to 120	14
Total	200

Building a Histogram...

Table 2.5 Frequency Distribution of the Long-Distance Bills in Example 2.4

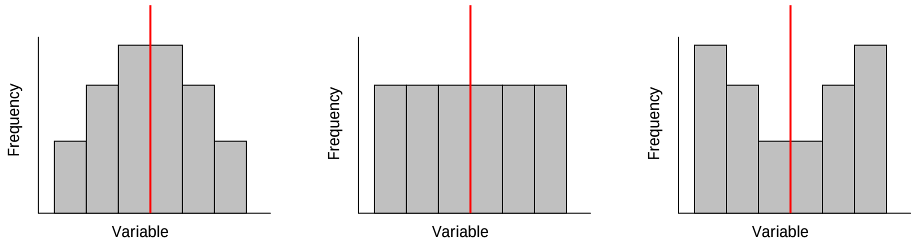
Class Limits	Frequency
0 to 15	71
15 to 30	37
30 to 45	13
45 to 60	9
60 to 75	10
75 to 90	18
90 to 105	28
105 to 120	14
Total	200



Shapes of Histograms...

Symmetry

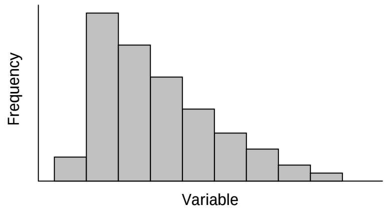
A histogram is said to be ***symmetric*** if, when we draw a **vertical line** down the center of the histogram, the two sides are identical in shape and size:



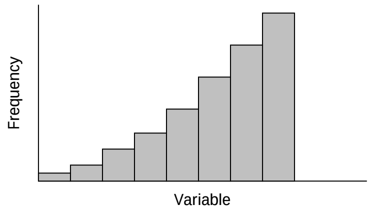
Shapes of Histograms…

Skewness

A skewed histogram is one with a long tail extending to either the right or the left:



Positively Skewed



Negatively Skewed

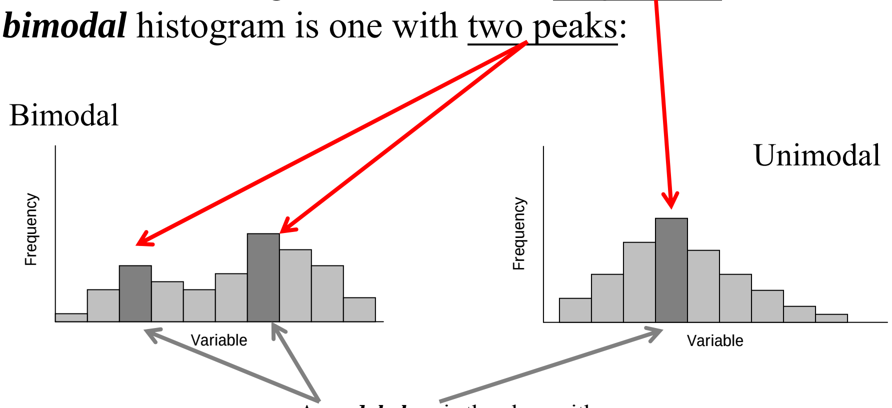
Shapes of Histograms…

Modality

A ***unimodal*** histogram is one with a single peak, while a ***bimodal*** histogram is one with two peaks:

Bimodal

Unimodal

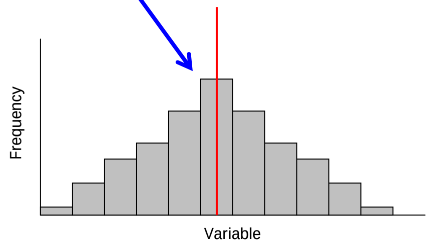


A ***modal class*** is the class with the largest number of observations

Shapes of Histograms…

Bell Shape

A special type of ***symmetric*** ***unimodal*** histogram is one that is bell shaped:



Bell Shaped

Many statistical techniques require that the population be bell shaped.

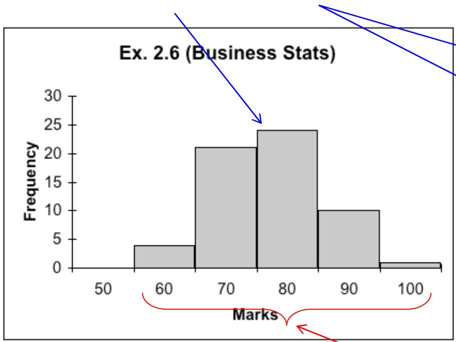
Drawing the histogram helps verify the shape of the population in question.

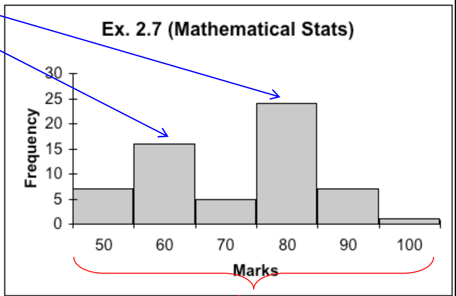
Histogram Comparison…

Compare & contrast the following histograms based on data from Ex. 2.6 & Ex. 2.7:

The two courses, Business Statistics and Mathematical Statistics have very different histograms…

unimodal vs. bimodal





spread of the marks (narrower | wider)

Stem & Leaf Display...

- Retains information about individual observations that would normally be lost in the creation of a histogram.

- Split each observation into two parts, a ***stem*** and a ***leaf***:

- e.g. Observation value: **42.19**
- There are several ways to split it up…

- We could split it at the decimal point:

- Or split it at the "tens" position (while rounding to the nearest integer in the "ones" position)

Stem	Leaf
42	19
4	2

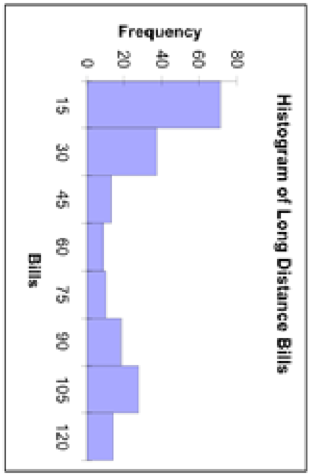
Stem & Leaf Display...

- Continue this process for all the observations. Then, use the "stems" for the classes and each leaf becomes part of the histogram (based on Example 2.4 data) as follows…

Stem	Leaf
0	00000000001111122222233333455555566666667788889999999
1	000001111233333334455555667889999
2	0000111112344666778999
3	001335589
4	124445589
5	33566
6	3458
7	022224556789
8	334457889999
9	00112222233344555999
10	001344446699
11	124557889

Thus, we still have access to our original data point's **value**!

Histogram and Stem & Leaf…



Stem	Leaf
0	00000000001111122222223333345555556666666778888999999
1	000001111233333334455555667889999
2	0000111112344666778999
3	001335589
4	124445589
5	33566
6	3458
7	022224556789
8	334457889999
9	00112222233344555999
10	001344446699
11	124557889

Compare the overall ***shapes*** of the figures…

Ogive…

- (pronounced "Oh-jive") is a graph of a ***cumulative frequency distribution***.
- We create an ogive in three steps…
- First, from the frequency distribution created earlier, calculate ***relative frequencies***:
- $$\text{Relative Frequency} = \frac{\text{\# of observations in a class}}{\text{Total \# of observations}}$$

Relative Frequencies...

- For example, we had 71 observations in our first class (telephone bills from \$0.00 to \$15.00). Thus, the relative frequency for this class is 71 ÷ 200 (the total # of phone bills) = 0.355 (or 35.5%)

Table 2.7 Relative Frequency Distribution for Example 2.4

Class Limits	Relative Frequency
0 to 15	71/200 = .355
15 to 30	37/200 = .185
30 to 45	13/200 = .065
45 to 60	9/200 = .045
60 to 75	10/200 = .050
75 to 90	18/200 = .090
90 to 105	28/200 = .140
105 to 120	14/200 = .070
Total	200/200 = 1.0

Ogive...

- Is a graph of a ***cumulative frequency distribution***.

- We create an ogive in three steps…
- 1) Calculate relative frequencies. ✓
- 2) Calculate ***cumulative relative frequencies*** by adding the current class' relative frequency to the previous class' cumulative relative frequency.
- (For the first class, its cumulative relative frequency is just its relative frequency)

Cumulative Relative Frequencies…

Table 2.8 Cumulative Relative Frequency Distribution for Example 2.4

Class Limits	Relative Frequency	Cumulative Relative Frequency
0 to 15	71/200 = .355	71/200 = .355
15 to 30	37/200 = .185	108/200 = .540
30 to 45	13/200 = .065	121/200 = .605
45 to 60	9/200 = .045	130/200 = .650
60 to 75	10/200 = .05	140/200 = .700
75 to 90	18/200 = .09	158/200 = .790
90 to 105	28/200 = .14	186/200 = .930
105 to 120	14/200 = .07	200/200 = 1.00

first class…

next class: .355+.185=.540

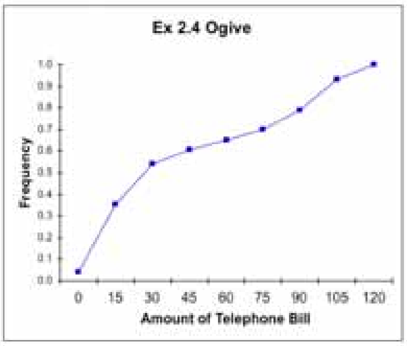
:

:

last class: .930+.070=1.00

Ogive…

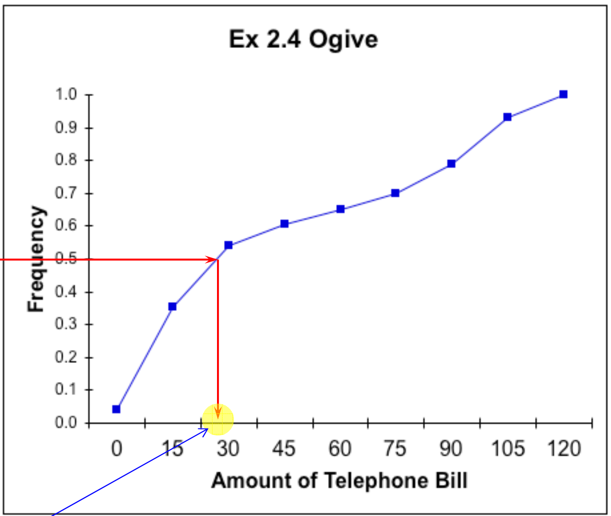
- Is a graph of a ***cumulative frequency distribution***.
- 1) Calculate relative frequencies. ✓
- 2) Calculate cumulative relative frequencies. ✓
- 3) Graph the cumulative relative frequencies…



Ogive...

The ogive can be used to answer questions like:

What telephone bill value is at the 50th percentile?



"around $35"

(Refer also to Fig. 2.13 in your textbook)

Describing Time Series Data

Observations measured at the same point in time are called ***cross-sectional*** data.

Observations measured at successive points in time are called ***time-series*** data.

Time-series data graphed on a ***line chart***, which plots the value of the variable on the vertical axis against the time periods on the horizontal axis.

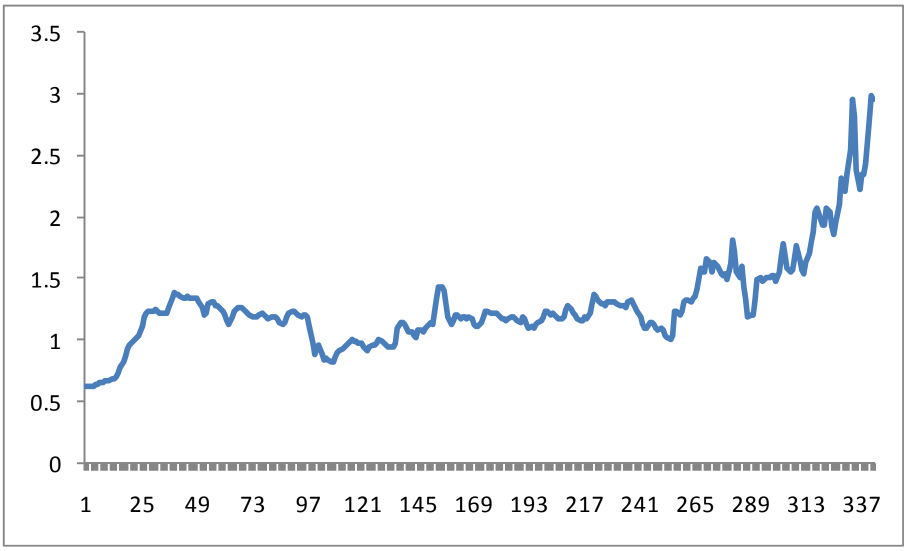
Example 2.8

We recorded the monthly average retail price of gasoline since 1978.

Xm02-08

Draw a line chart to describe these data and briefly describe the results.

Example 2.8

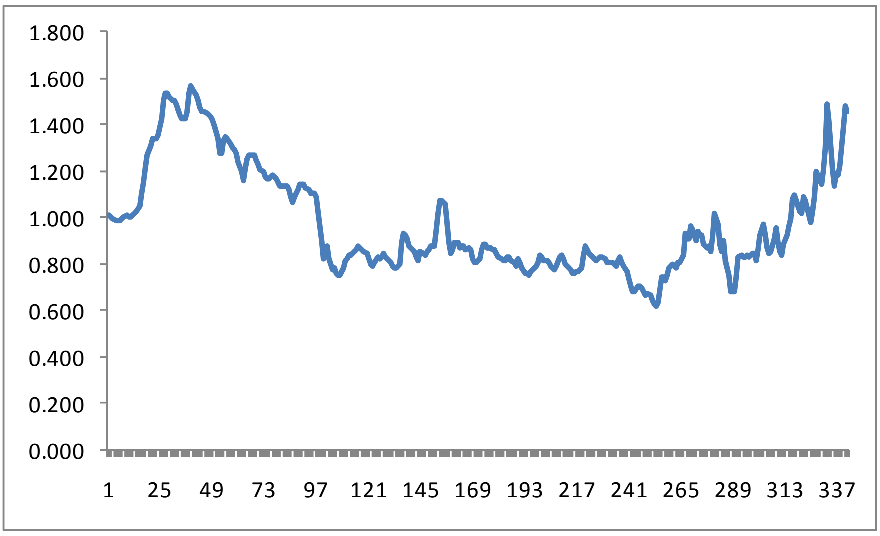


Example 2.9 Price of Gasoline in 1982-84 Constant Dollars

Xm02-09

Remove the effect of inflation in Example 2.8 to determine whether gasoline prices are higher than they have been in the past after removing the effect of inflation.

Example 2.9



Relationship between Two Nominal Variables...

So far we've looked at tabular and graphical techniques for one variable (either nominal or interval data).

A ***cross-classification table (***or cross-tabulation table) is used to describe the relationship between **two** nominal variables.

A cross-classification table lists the ***frequency*** of ***each combination*** of the values of the two variables…

Example 2.10

In a major North American city there are four competing newspapers: the Post, Globe and Mail, Sun, and Star.

To help design advertising campaigns, the advertising managers of the newspapers need to know which segments of the newspaper market are reading their papers.

A survey was conducted to analyze the relationship between newspapers read and occupation.

Example 2.10

A sample of newspaper readers was asked to report which newspaper they read: Globe and Mail (1) Post (2), Star (3), Sun (4), and to indicate whether they were blue-collar worker (1), white-collar worker (2), or professional (3).

The responses are stored in file Xm02-10.

Example 2.10

By counting the number of times each of the 12 combinations occurs, we produced the Table 2.9.

	Occupation			
Newspaper	Blue Collar	White Collar	Professional	Total
G&M	27	29	33	89
Post	18	43	51	112
Star	38	21	22	81
Sun	37	15	20	72
Total	120	108	126	354

Example 2.10

If occupation and newspaper are related, then there will be differences in the newspapers read among the occupations. An easy way to see this is to covert the frequencies in each column to relative frequencies in each column. That is, compute the column totals and divide each frequency by its column total.

		Occupation	
Newspaper	Blue Collar	White Collar	Professional
G&M	27/120 =.23	29/108 = .27	33/126 = .26
Post	18/120 = .15	43/108 = .40	51/126 = .40
Star	38/120 = .32	21/108 = .19	22/126 = .17
Sun	37/120 = .31	15/108 = .14	20/126 = .16

Example 2.10

Interpretation: The relative frequencies in the columns 2 & 3 are similar, but there are large differences between columns 1 and 2 and between columns 1 and 3.

Table 2.10 Column Relative Frequencies for Example 2.8

		Occupation	
Newspaper	Blue Collar	White Collar	Professional
G&M	27/120 =.23	29/108 = .27	33/126 = .26
Post	18/120 = .15	43/108 = .40	51/126 = .40
Star	38/120 = .32	21/108 = .19	22/126 = .17
Sun	37/120 = .31	15/108 = .14	20/126 = .16

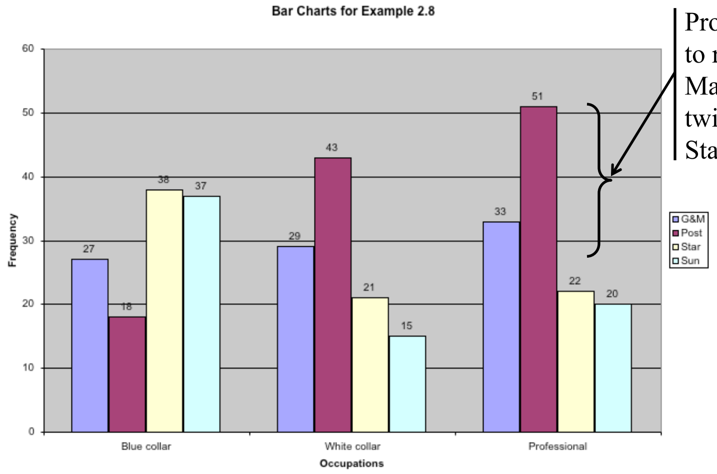
similar

dissimilar

This tells us that blue collar workers tend to read different newspapers from both white collar workers and professionals and that white collar and professionals are quite similar in their newspaper choice.

Graphing the Relationship Between Two Nominal Variables…

Use the data from the cross-classification table to create bar charts…



Professionals tend to read the Globe & Mail more than twice as often as the Star or Sun…

Graphing the Relationship Between Two **Interval** Variables…

Moving from nominal data to interval data, we are frequently interested in how two interval variables are related.

To explore this relationship, we employ a ***scatter diagram***, which plots two variables against one another.

The ***independent*** variable is labeled X and is usually placed on the horizontal axis, while the other, ***dependent*** variable, Y, is mapped to the vertical axis.

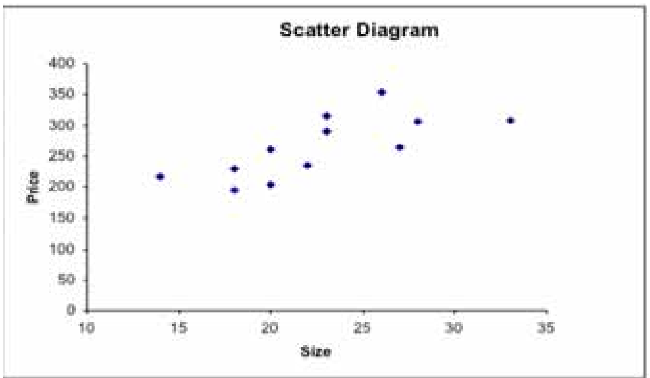
Example 2.12

A real estate agent wanted to know to what extent the selling price of a home is related to its size. To acquire this information he took a sample of 12 homes that had recently sold, recording the price in thousands of dollars and the size in hundreds of square feet. These data are listed in the accompanying table. Use a graphical technique to describe the relationship between size and price. Xm02-12

Size	23	18	26	20	22	14	33	28	23	20	27	18
Price	315	229	355	261	234	216	308	306	289	204	265	195

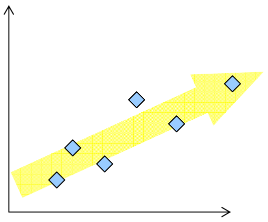
Example 2.12

It appears that in fact there is a relationship, that is, the greater the house size the greater the selling price…

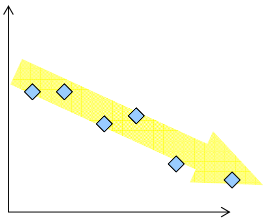


Patterns of Scatter Diagrams…

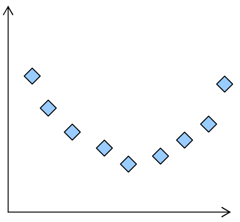
Linearity and Direction are two concepts we are interested in



Positive Linear Relationship



Negative Linear Relationship



Weak or Non-Linear Relationship

Summary I…

Factors That Identify When to Use Frequency and Relative Frequency Tables, Bar and Pie Charts

1. Objective: Describe a single set of data.
2. Data type: Nominal

Factors That Identify When to Use a Histogram, Ogive, or Stem-and-Leaf Display

1. Objective: Describe a single set of data.
2. Data type: Interval

Factors that Identify When to Use a Cross-classification Table

1. Objective: Describe the relationship between two variables.
2. Data type: Nominal

Factors that Identify When to Use a Scatter Diagram

1. Objective: Describe the relationship between two variables.
2. Data type: Interval

Summary II...

	Interval Data	Nominal Data
Single Set of Data	Histogram, Ogive, Stem-and-Leaf Display	Frequency and Relative Frequency Tables, Bar and Pie Charts
Relationship Between Two Variables	Scatter Diagram	Cross-classification Table, Bar Charts