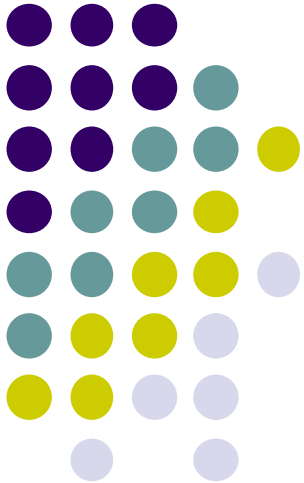
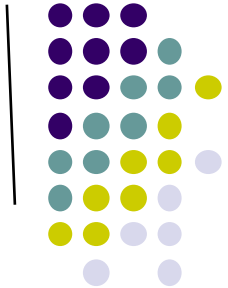
Ch 11 實習 (1)



Agenda



- 何謂假設檢定
  - C11使用的時機
- 主要檢定的四步驟
  - 信賴區間法
  - 假設檢定法（Rejection region or critical value)
  - P-value判別法
- a, B意涵
  - B 如何計算 （下一次講）

一、何謂假設檢定

- 所謂假設檢定，係指在尚未蒐集樣本資料、進行推論之前，就事先對母體的某種特徵性質作一合理的假設敘述，再利用隨機抽出的樣本及抽樣分配，配合機率原理，以判斷此項假設是否為真

- 若抽出的樣本資料與所陳述的假設很不一致，檢定的結果必然認為這個假設不對，則否定或拒絕（reject）這個假設

一、何謂假設檢定

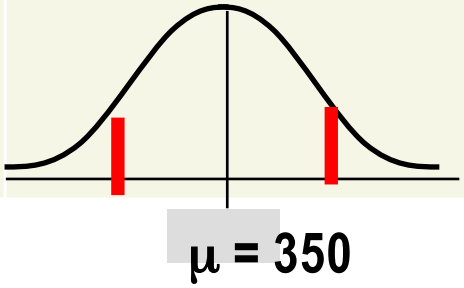
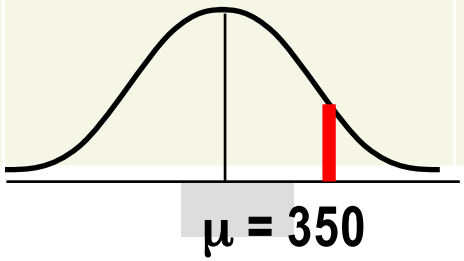
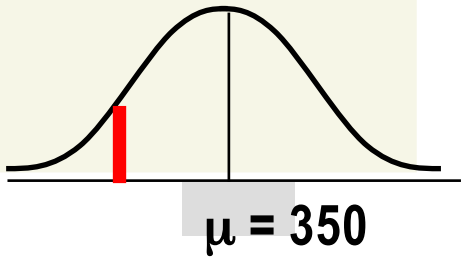
- 若抽出的樣本資料與所陳述的假設不會很不一致，檢定的結果就沒有充分理由斷定這個假設不對，但也不認為這個假設是對的
- 假設檢定的主要精神在於尋找證據來拒絕H_0，所以假設檢定只能檢定H_0是否顯著錯誤，而不能判斷其絕對正確

一、C11使用的時機

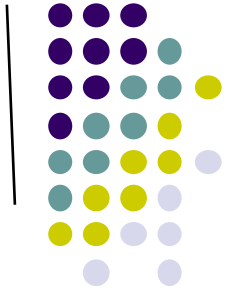
- 母體的μ, σ^2 已知
- 且假設σ^2不會改變的情況
- 我們才能用 Z score

二、主要檢定的四步驟

三種情況都一樣(H0: u =0)

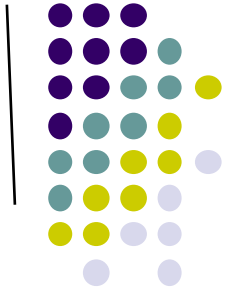
	$H1: u \neq 0$	$H1: u > 0$	$H1: u < 0$
S1:決定假設(單尾或雙尾)	雙尾	單尾(右尾)	單尾(左尾)
S2: 決定critical point (value)	$\pm Z_{\frac{a}{2}}$	Z_a	$-Z_a$
S3:算 Test statistic	$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
S4:判定與給結論			

Concepts of Hypothesis Testing



- The critical concepts of hypothesis testing.
  - Example:
    - An operation manager needs to determine if the mean demand during lead time is greater than 350.
    - If so, changes in the ordering policy are needed.
  - There are two hypotheses about a population mean:
    - H_0: The null hypothesis $\mu = 350$ （等號一定放這）
    - H_1: The alternative hypothesis $\mu > 350$ （題目問的）

Example 1



- A spouse suspects that the average amount of money spent on Christmas gifts for immediate family members is above $1,200. The correct set of hypotheses is:

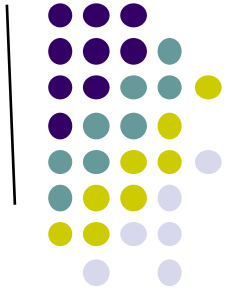
  a. $H0: \mu = 1200$ vs. $H1: \mu < 1200$

  b. $H0: \mu > 1200$ vs. $H1: \mu = 1200$

  c. $H0: \mu = 1200$ vs. $H1: \mu > 1200$

  d. $H0: \mu < 1200$ vs. $H1: \mu = 1200$

Example 1



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  b. $H0: \mu > 1200$ vs. $H1: \mu = 1200$

  c. $H0: \mu = 1200$ vs. $H1: \mu > 1200$

  d. $H0: \mu < 1200$ vs. $H1: \mu = 1200$

Example 1、信賴區間檢定法

A new billing system for a department store will be cost- effective only if the mean monthly account is more than $170. A sample of 400 accounts has a mean of $178.

If accounts are approximately normally distributed with σ = $65, can we conclude that the new system will be cost effective?

Stpe1: 設立假設

H0: u =170
H1: u>170

Stpe3: 判定結果

- Reject H0 in favor H1
- 因為ＣＩ不包含 170, 所以我們有足夠證據拒絕他的宣稱

Stpe2: 求信賴區間 (一定是雙尾）

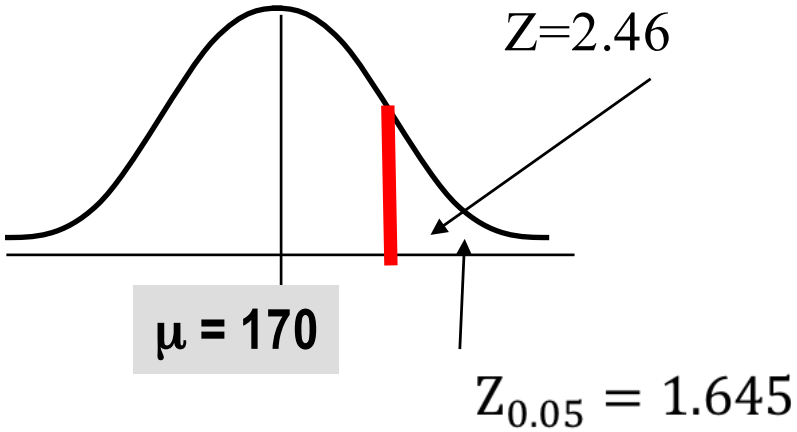
$$\bar{X} \pm Z_{a/2} * \frac{\sigma}{\sqrt{n}} = 178 \pm 1.96 * \frac{65}{20} = (171.63 , 184,37)$$

Example 1、Rejection Region/critical point 法

A new billing system for a department store will be cost- effective only if the mean monthly account is more than $170. A sample of 400 accounts has a mean of $178.

If accounts are approximately normally distributed with σ = $65, can we conclude that the new system will be cost effective?

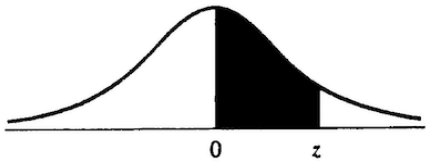
- Step1: 設定假設
  - Ho: u=170, H1: u>170
- Step2:計算critical point (value)
  - $Z_{0.05} = 1.645$
- Step3: 算統計量
  - $Z=\frac{\bar{X}-u}{\sigma/\sqrt{n}}=\frac{178-170}{65/20}=2.46$
- Step4: Don't reject Ho



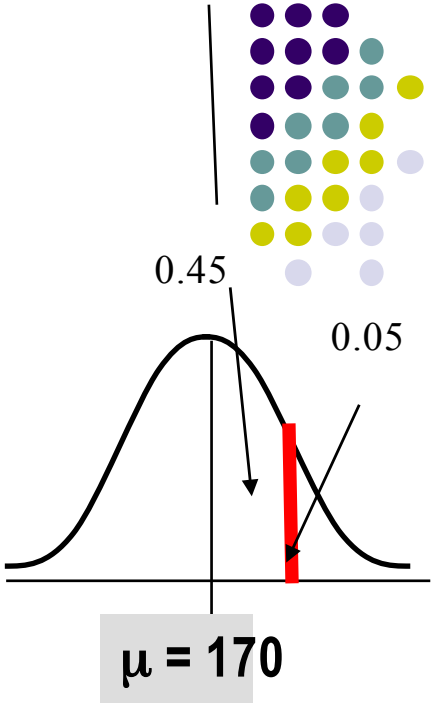
Since Z = 2.46 > 1.645, reject the null hypothesis, in favor of the alternative hypothesis.

Normal Table — Areas of the Standard Normal Distribution

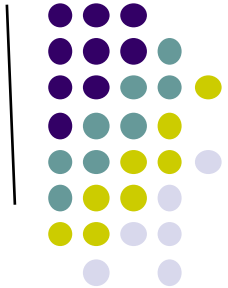
The entries in this table are the probabilities that a random variable with a standard normal distribution assumes a value between 0 and z; the probability is represented by the shaded area under the curve in the accompanying figure. Areas for negative values of z are obtained by symmetry.



	Second Decimal Place in z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990



P-value Method



- The p-value provides information about the amount of statistical evidence that supports the alternative hypothesis.

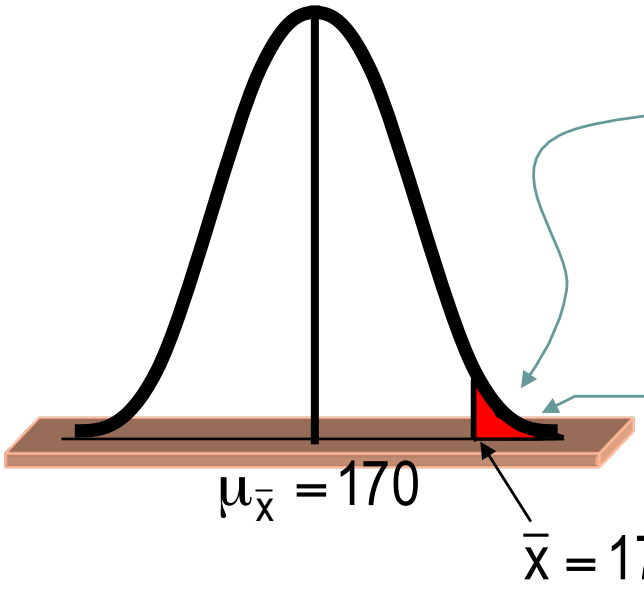
  – The p-value of a test is the probability of observing a test statistic at least as extreme as the one computed, given that the null hypothesis is true.

–虛無假設成立，獲得檢定統計量及更極端數值之機率。

–Let us demonstrate the concept on Example

P-value Method

The probability of observing a test statistic at least as extreme as 178, given that $\mu = 170$ is…



$$P(\bar{x} > 178 \text{ when } \mu = 170)$$

$$= P\left(z > \frac{178 - 170}{65/\sqrt{400}}\right)$$

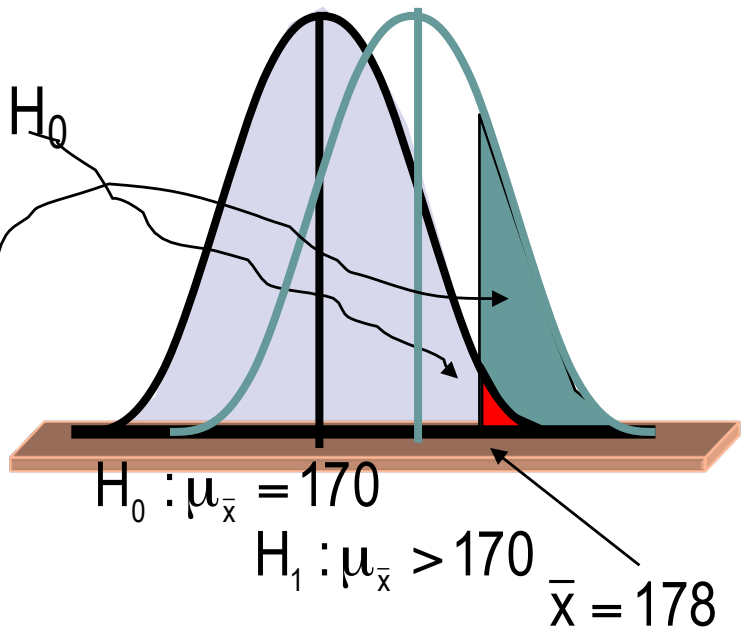
$$= P(z > 2.4615) = .0069$$

The p-value

Interpreting the p-value

Because the probability that the sample mean will assume a value of more than 178 when $\mu = 170$ is so small (.0069), there are reasons to believe that $\mu >$ 170. (找到比178更大的值的機率）

Note how the event $\bar{x} > 178$ is rare under H_0 when $\mu_{\bar{x}} = 170$, but... ...it becomes more probable under H_1, when $\mu_{\bar{x}} > 170$

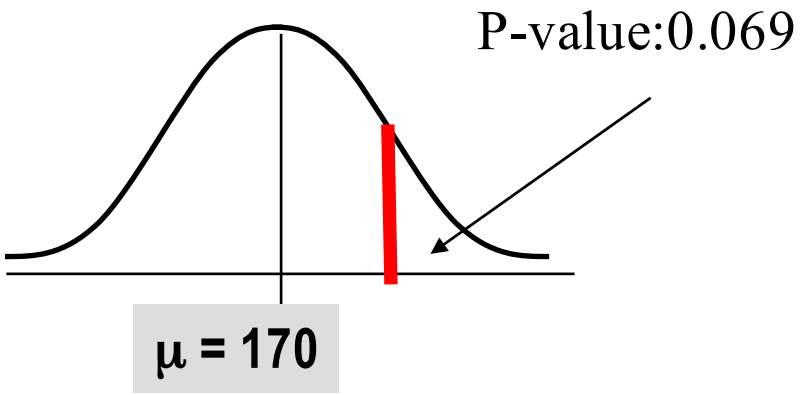


Example 1、 p-value 判定法

A new billing system for a department store will be cost- effective only if the mean monthly account is more than $170. A sample of 400 accounts has a mean of $178.

If accounts are approximately normally distributed with σ = $65, can we conclude that the new system will be cost effective?

- Step1: 設定假設
  - Ho: u=170, H1: u>170
- Step2:計算critical point (value)
  - $Z_{0.05} = 1.645$
- Step3: 算 p-value
  - $Z=\frac{\bar{X}-u}{\sigma/\sqrt{n}}=\frac{178-170}{65/20}=2.46$
  - P(Z>2.46)=0.0069
- Step4: Don't reject Ho



Since P-value <0.05, reject the null hypothesis, in favor of the alternative hypothesis.

Normal Table Areas of the Standard Normal Distribution

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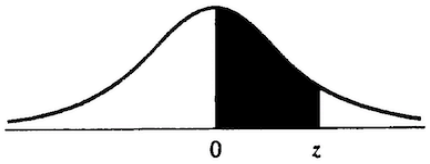

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0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
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1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
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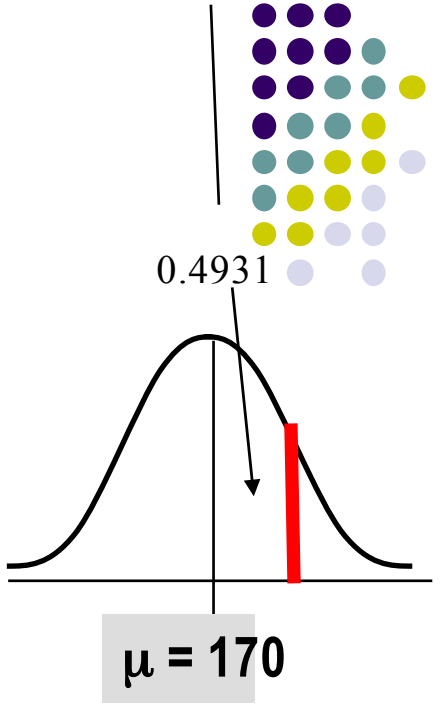
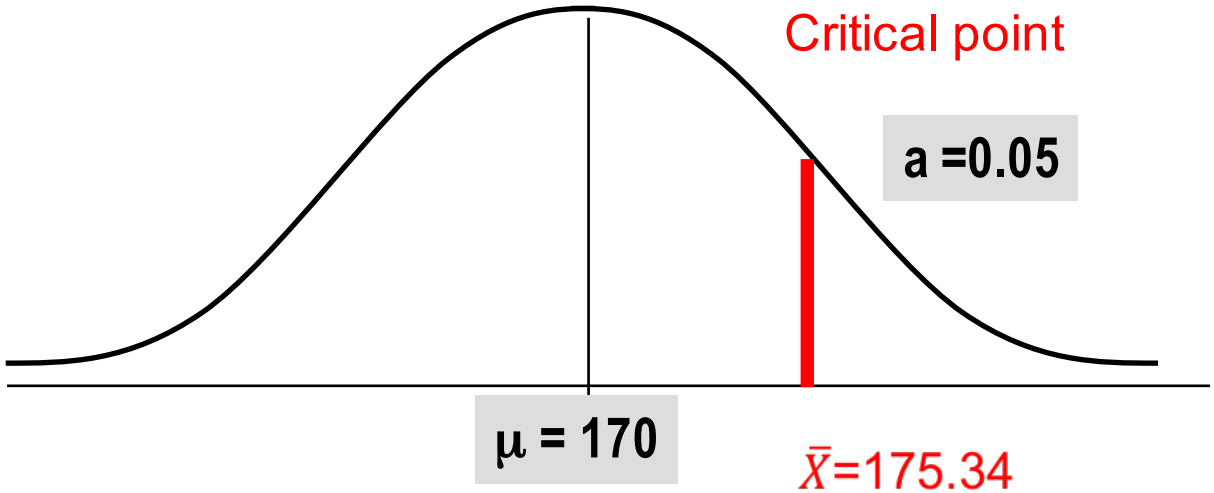
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990



Z=2.46
P(Z>2.46)
=0.5-0.4931
=0.0069

老師上課說的....



	欲求解	比較點
Threshold $\bar{X}$	$\bar{X}$=178（題目給的）	$\bar{X}$=175.34
Critical point（Z）	Z score=2.46（計算）	$Z_{a=0.05} = 1.645$ (查表）
P-value (機率）	P(Z > Z score)= 0.069	a =0.05（題目給的）

Example 2、信賴區間檢定法

例一： 某公司保證其所生產的電暖器五年內免費修理。現在公司發現顧客在五年內要求修理的次數相對提高，因此，想知道該電器產品的保用年限是否小於五年。為了重新檢定電暖器的保用年限，於是隨機搜查36個顧客所購買的電暖器的資料，設電暖器的壽命的$\bar{X}=4.8$，$\sigma=1.5$年。試回答下列問題

Stpe1: 設立假設

H0: u =5
H1: u<5

Stpe2: 求信賴區間 (一定是雙尾）

$$\bar{X} \pm Z_{a/2} * \frac{\sigma}{\sqrt{n}} = 4.8 \pm 1.96 * \frac{1.5}{6} = (4.31, 5.29)$$

Stpe3: 判定結果

- Do not reject H0
- 因為ＣＩ包含 5, 所以我們沒有足夠證據拒絕他的宣稱

Example 2、Rejection Region/critical point 法

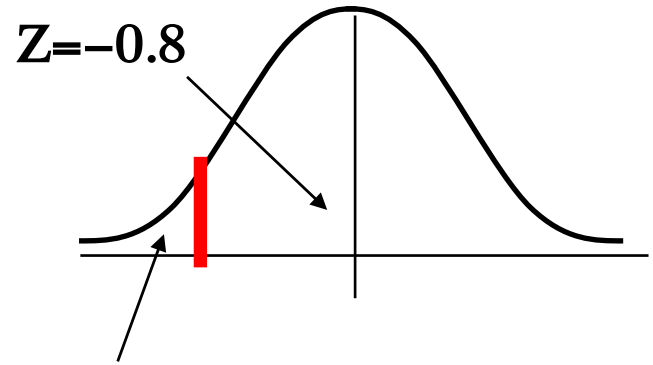
例一： 某公司保證其所生產的電暖器五年內免費修理。現在公司發現顧客在五年內要求修理的次數相對提高，因此，想知道該電器產品的保用年限是否小於五年。為了重新檢定電暖器的保用年限，於是隨機搜查36個顧客所購買的電暖器的資料，設電暖器的壽命的$\bar{X}=4.8$，$\sigma=1.5$年。試回答下列問題

Step 1:設定假說
- H0=5, H1<5

Step 1:計算critical point
- $Z_{0.05}=-1.645$

Step3:算統計量
- $Z=\frac{\bar{X}-u}{\sigma/\sqrt{n}}=\frac{4.8-5}{1.5/6}=-0.8$

Step 4: Do not reject Ho



$Z_{0.05}=-1.645$

Do not reject H0
-0.8 > -1.645

Example 2、P-value 判定法

例一： 某公司保證其所生產的電暖器五年內免費修理。現在公司發現顧客在五年內要求修理的次數相對提高，因此，想知道該電器產品的保用年限是否小於五年。為了重新檢定電暖器的保用年限，於是隨機搜查36個顧客所購買的電暖器的資料，設電暖器的壽命的$\bar{X}=4.8$，$\sigma=1.5$年。試回答下列問題

Step 1:設定假說

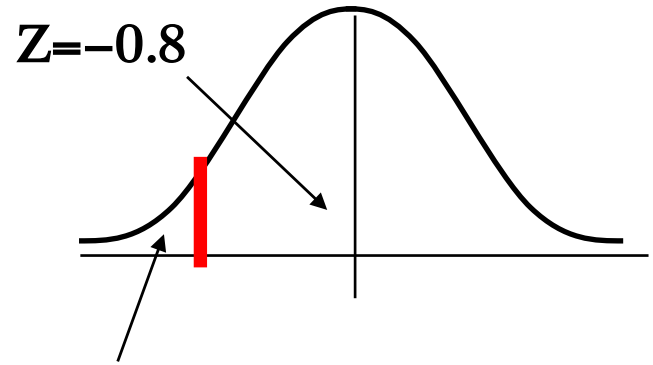
- H0=5, H1<5

Step 1:計算critical point

- $Z_{0.05}=-1.645$

Step3: 算p-value

- Z= $\frac{\bar{X}-u}{\sigma/\sqrt{n}}=\frac{4.8-5}{1.5/6}=-0.8$
- P(Z<-0.8)=0.5-0.2881=0.2009

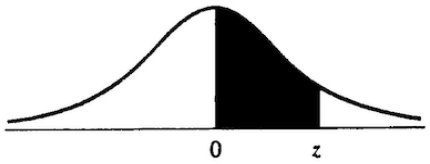


$Z_{0.05}=-1.645$

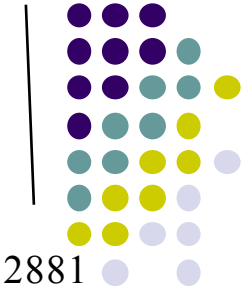
Do not reject H0
0.2009 > 0.05

Normal Table Areas of the Standard Normal Distribution

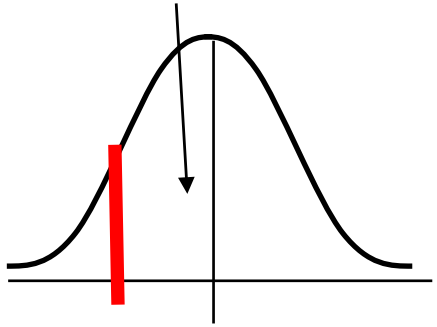
The entries in this table are the probabilities that a random variable with a standard normal distribution assumes a value between 0 and z; the probability is represented by the shaded area under the curve in the accompanying figure. Areas for negative values of z are obtained by symmetry.



	Second Decimal Place in z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990



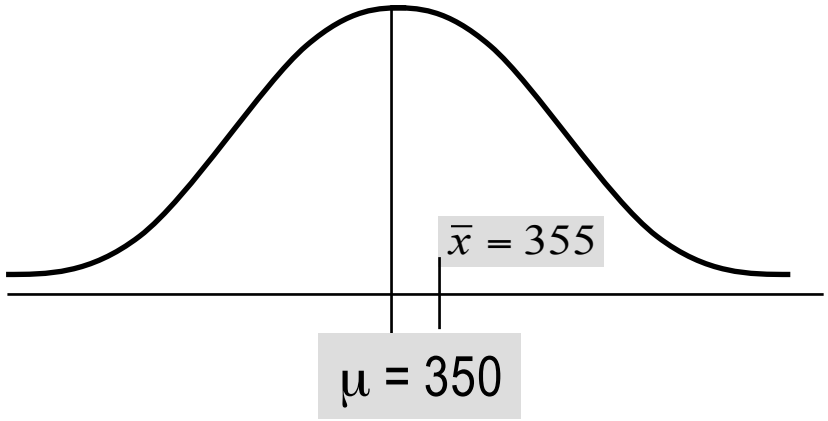
0.2881



Z=-0.8
P(Z<-0.8)
=0.5-0.2881
=0.2009

三、a, B意涵

- Assume the null hypothesis is true (μ= 350).



Two types of errors may occur when deciding whether to reject H_0 based on the statistic value.

- Type I error: Reject H_0 when it is true.
- Type II error: Do not reject H_0 when it is false.

Example continued

- Type I error: Reject H_0 (μ = 350) in favor of H_1 (μ > 350) when the real value of μ is 350.
- Type II error: Believe that H_0 is correct (μ = 350) when the real value of μ is greater than 350.

三、a, B意涵

	Null True	Null False
Fail to reject null	Correct Decision (Significant Level: 1-a)	Type II error (β)
Reject null	Type I error (α)	Correct Decision (Power=1-B)

在檢定內，希望power越大越好。表示當Ho是錯的，能有效拒絕他