Ch 8 實習

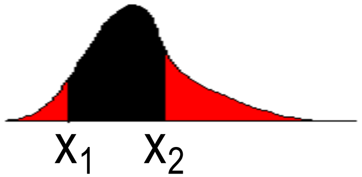
Agenda

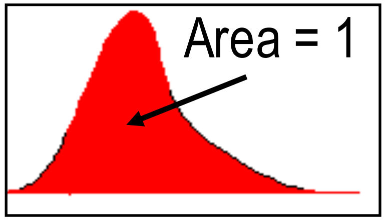
- 學習目標:
  - 知道某分配，要會求u, sd, 某一段區間的機率
- 了解連續機率分配
- 了解不同分配，如何運算
  - Uniform Distribution
  - Normal Distribution
  - Standard Normal Distribution
  - Exponential Distribution
- 計算機計算(Poisson, Exponential Distribution)
- 查表 (常態分配)與EXCEL的運用
- Project解說

一、了解連續機率分配

- To calculate probabilities we define a probability density function $f(x)$→pdf
- The density function satisfies the following conditions
  - $f(x)$ is non-negative,
  - The total area under the curve representing $f(x)$ equals 1.

$P(x_1 <= X <= x_2)$ ⟶





●不連續機率分配時, f(x)是自然界的現象，可以解釋。但是連續機率分配時， f(x)是數學工具，不能解釋。我們有興趣的是它構成的面積。

- The probability that X falls between x_1 and x_2 is found by calculating the area under the graph of $f(x)$ between x_1 and x_2.
- $\int_{x1}^{x2} X.f(x)\,dx$ →透過積分可以求得面積

二、了解不同分配，如何運算
Uniform Distribution (均勻分配)

- A random variable **X** is said to be uniformly distributed if its density function is

$$f(x) = \frac{1}{b-a} \quad a \le x \le b.$$

(a) 為什麼f(x)是 1/(b-a)？

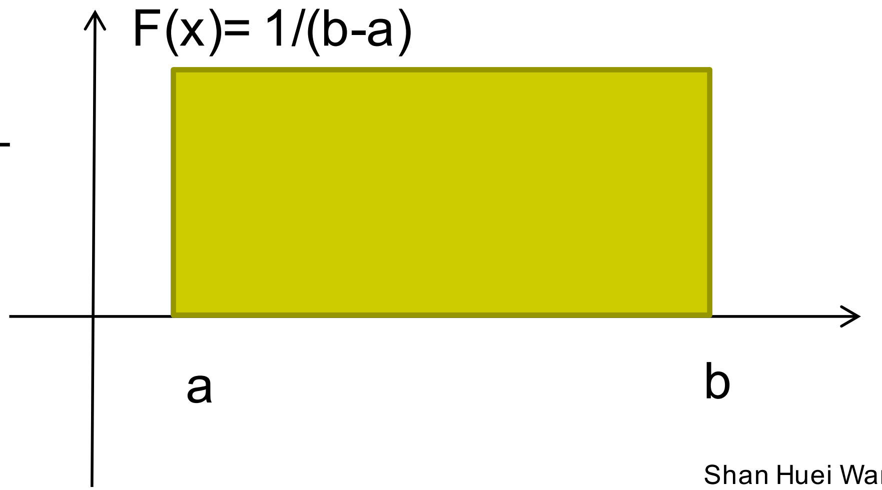
因為長方形面積為長＊寬（機率為 1）。

(b) f(x)=1/(b-a) ,意涵為何？

表示表示任何一點，f(x)皆為1/(b-a)

- The expected value and the variance are

$$E(X) = \frac{a+b}{2} \qquad V(X) = \frac{(b-a)^2}{12}$$



Example 1 （類似1)

- The weekly output of a steel mill is a uniformly distributed random variable that lies between 110 and 175 metric tons.

a. Compute the probability that the steel mill will produce more than 150 metric tons next week.

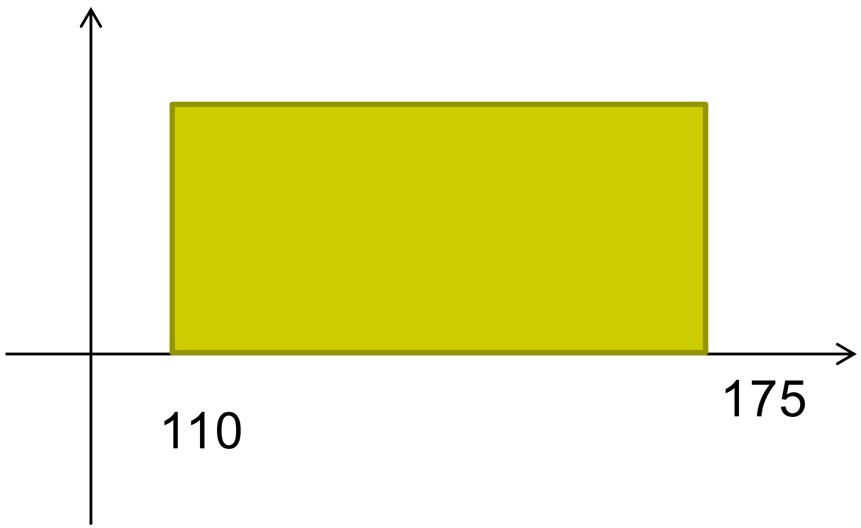
b. Determine the probability that the steel mill will produce between 120 and 160 metric tons next week.

Step 1:畫出圖形
Step2:寫出 f(x)
Step3: 求出題目的面積

Example 1 （類似1)

- The weekly output of a steel mill is a uniformly distributed random variable that lies between 110 and 175 metric tons.

Step 1:畫出圖形
Step2:寫出 f(x)
Step3: 求出題目的面積



$$f(x) = \frac{1}{(175-110)} = \frac{1}{65}, 110 \leqq x \leqq 175$$

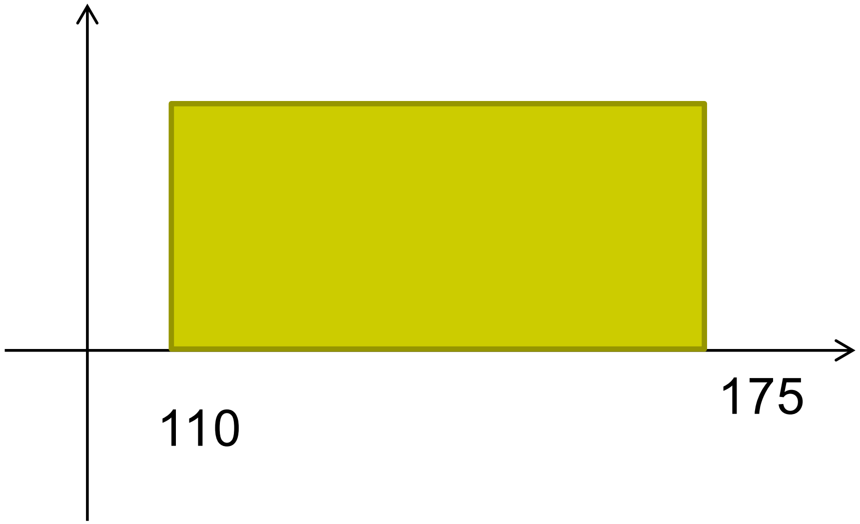
Example 1 （類似1)

- The weekly output of a steel mill is a uniformly distributed random variable that lies between 110 and 175 metric tons.

a. Compute the probability that the steel mill will produce more than 150 metric tons next week.

b. Determine the probability that the steel mill will produce between 120 and 160 metric tons next week.

Step3: 求出題目的面積



a. P(X ≧ 150) =

a. $P(X \geqq 150) = (175-150)\frac{1}{65} = 0.3846$

b. P(120 ≦ X ≦ 160)

b. $P(120 \leqq X \leqq 160) = (160-120)\frac{1}{65} = 0.6154$

Example 2

- The following function is the density function for the random variable X：

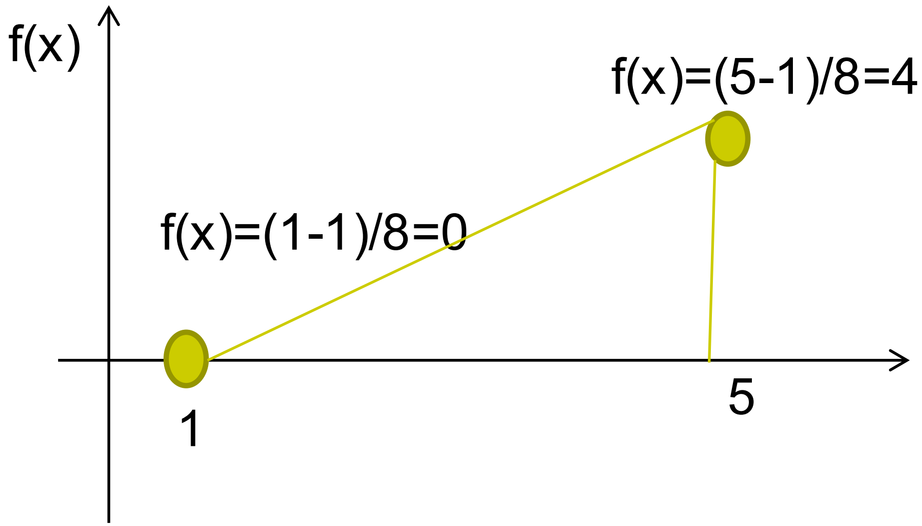
  f(x)=(x-1)/8, 1≦x ≦ 5

a. Graph the density function

b. Find the probability that X lies between 2 and 4

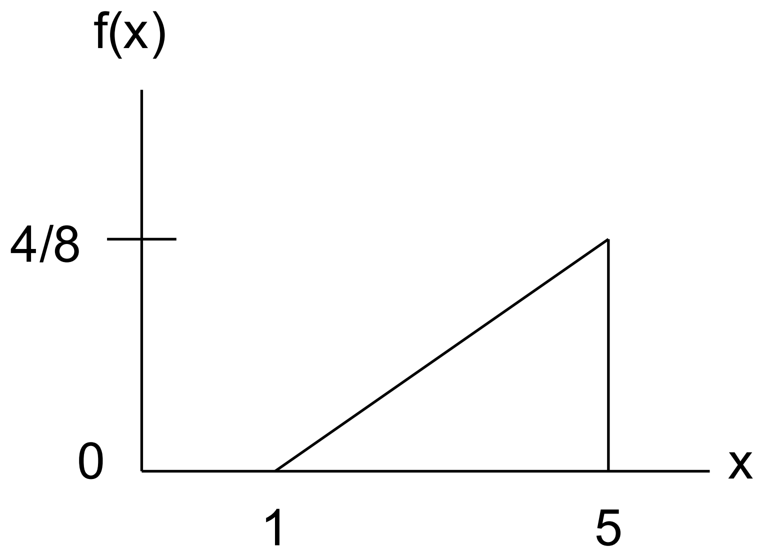
c. What is the probability that X is less than 3?

Step 1:畫出圖形
Step2:寫出 f(x)
Step3: 求出題目的面積



Solution

- a. *P.d.f*: $f(x)=(x-1)/8,\ 1 \leqq x \leqq 5$



Example 2

- The following function is the density function for the random variable X：

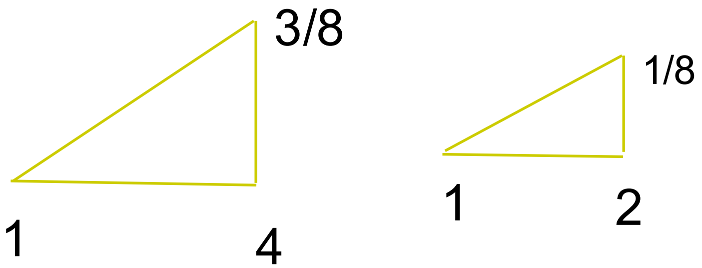
  $f(x)=(x-1)/8,\ 1 \leqq x \leqq 5$

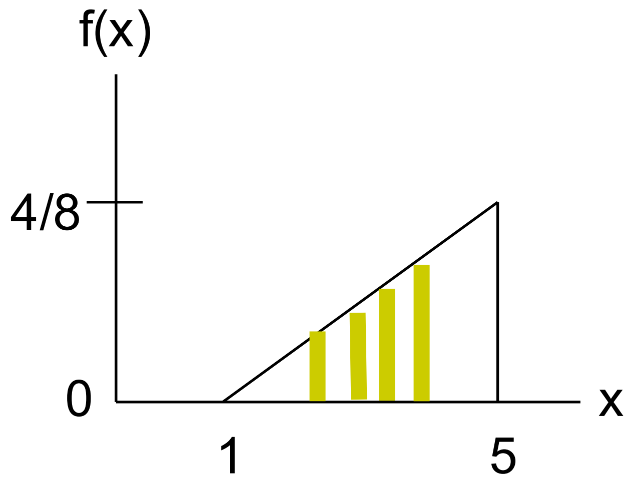
b. Find the probability that X lies between 2 and 4

Step3: 求出題目的面積

b. $P(2 < X < 4)$

$=P(X < 4) - P(X < 2)$
$=(0.5)(3/8)(4-1) - (.5)(1/8)(2-1) = .5625 - .0625 = .5$





Example 2

- The following function is the density function for the random variable X：

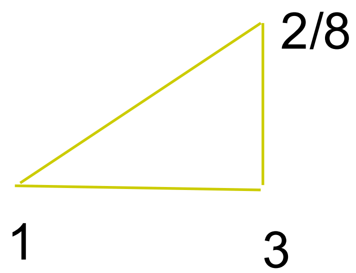
$$f(x)=(x-1)/8,\ 1 \leqq x \leqq 5$$

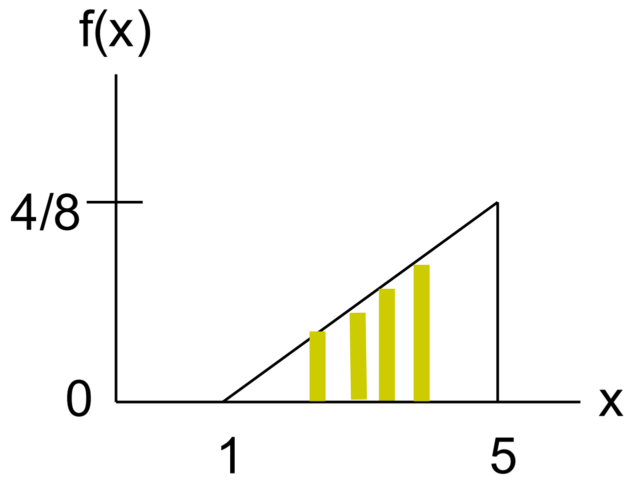
c. What is the probability that X is less than 3?

Step3: 求出題目的面積

c. $P(X < 3) =$

$P(X < 3) = (.5)(2/8)(3-1) = .25$





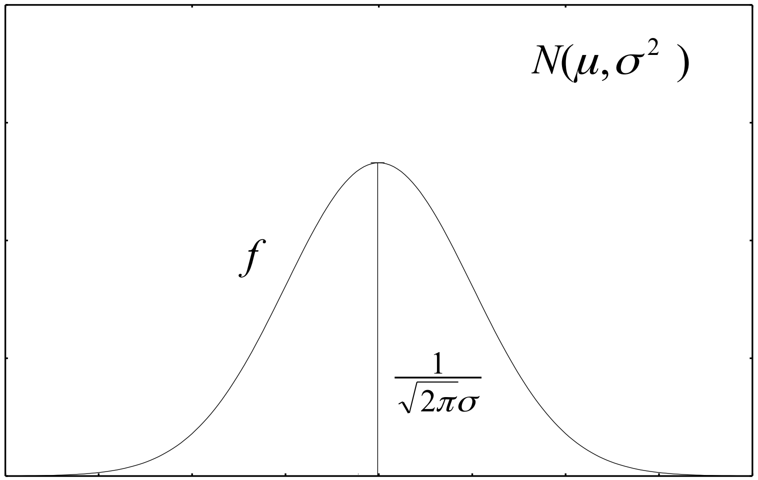
二、了解不同分配，如何運算

Normal Distribution (常態分配)

- A random variable **X** with mean μ and variance σ^2 is normally distributed if its probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty \le x \le \infty$$

$$where\ \pi = 3.14159\ldots \quad and \quad e = 2.71828\ldots$$



要求常態分配的面積，
請問可以積分嗎？

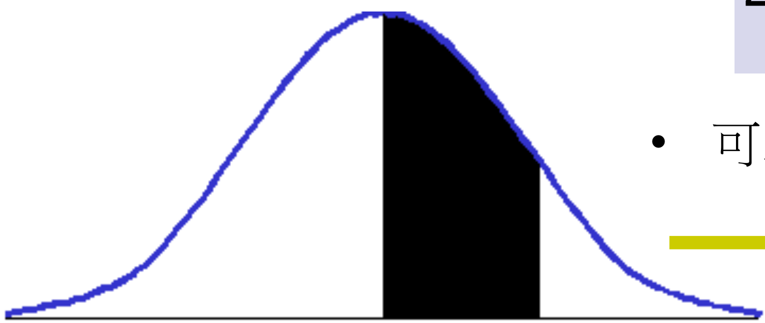
需要這麼辛苦嗎？

二、了解不同分配，如何運算
Normal Distribution (常態分配)

- Two facts help calculate normal probabilities:
  - The normal distribution is symmetrical.
  - Any normal distribution can be transformed into a specific normal distribution called…**"Standard Normal Distribution"**

Normalization (正規化)

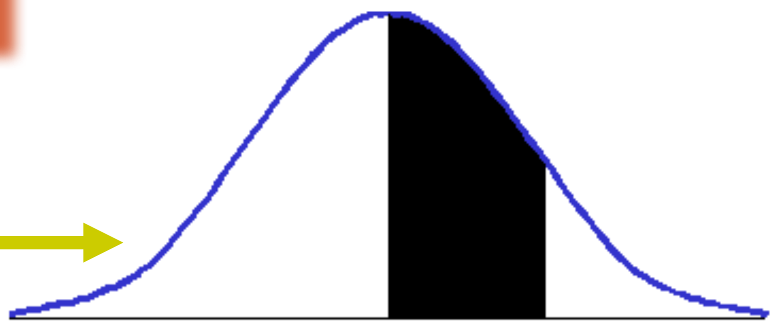
$N(\mu, \sigma)=N(25, 5)$



$$Z = \frac{X - \mu_x}{\sigma_x}$$

- 可以查表

$N(\mu, \sigma)=N(0, 1)$



二、Normal Distribution (類似2,3,6)

- The amount of time it takes to assemble a computer is normally distributed, with a mean of 50 minutes and a standard deviation of 10 minutes. What is the probability that a computer is assembled in a time between 45 and 60 minutes?

Step1: 列出題目
Step2:將題目正規化，求出 Z 值範圍
Step3: 查表求機率（或用excel)

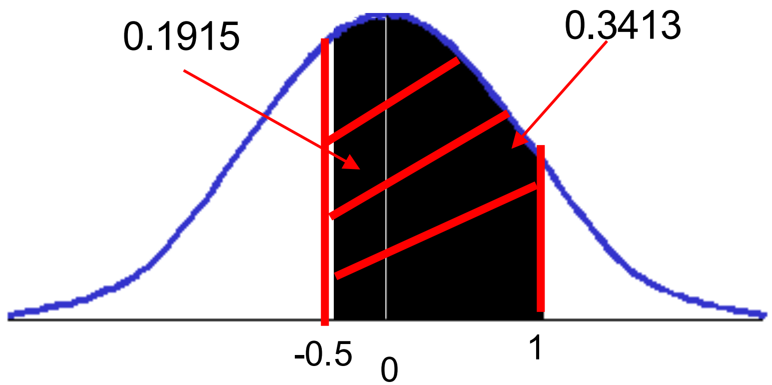
二、 Normal Distribution (常態分配)

- The amount of time it takes to assemble a computer is normally distributed, with a mean of 50 minutes and a standard deviation of 10 minutes. What is the probability that a computer is assembled in a time between 45 and 60 minutes?

Step1:列出題目

- P(45 ≦ X ≦ 60)

$N(\mu, \sigma)$=N(50, 10)



Step2:將題目正規化，求出 Z 值範圍

P(45 ≦ X ≦ 60) =

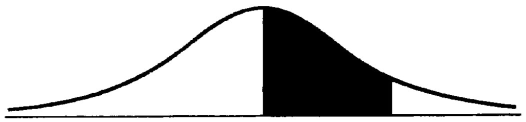
- $P\left(\frac{45-50}{10} \leq \frac{x-\mu}{\sigma} \leq \frac{45-50}{10}\right)$
- $P(-0.5 \leq Z \leq 1)$

Step3: 查表求機率

- $P(-0.5 \leq Z \leq 1)$=0.1915+0.3413=0.5328

Normal Table — Areas of the Standard Normal Distribution

The entries in this table are the probabilities that a random variable with a standard normal distribution assumes a value between 0 and z; the probability is represented by the shaded area under the curve in the accompanying figure. Areas for negative values of z are obtained by symmetry.



Second Decimal Place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999									
4.0	0.49997									
4.5	0.499997									

二、 Normal Distribution (常態分配)

- The amount of time it takes to assemble a computer is normally distributed, with a mean of 50 minutes and a standard deviation of 10 minutes. What is the probability that a computer is assembled in a time between 45 and 60 minutes?

Step1: 列出題目

- $P(45 \leqq X \leqq 60)$

Step2:將題目正規化，求出Ｚ值範圍

$P(45 \leqq X \leqq 60) =$

- $P\left(\frac{45-50}{10} \leq \frac{x-\mu}{\sigma} \leq \frac{45-50}{10}\right)$
- $P(-0.5 \leq Z \leq 1)$ $z_0 = -.5$ $z_0 = 1$

Step3: 查表求機率

- $P(-0.5 \leq Z \leq 1)=0.1915+0.3413=0.5328$

- 給X時，求機率（ＥＸＣＥＬ）
- =NORM.DIST(x,u,sd,1)
- =NORM.DIST(x,u,sd,0)

- $P(45 \leqq X \leqq 60)$
- =NORM.DIST(60,50,10,1)-
- NORM.DIST(45,50,10,1)
- =0.5328

HW# 3

- X is normally distributed with mean 300 and standard deviation 40. What value of X does only the top 15 % exceed?
-

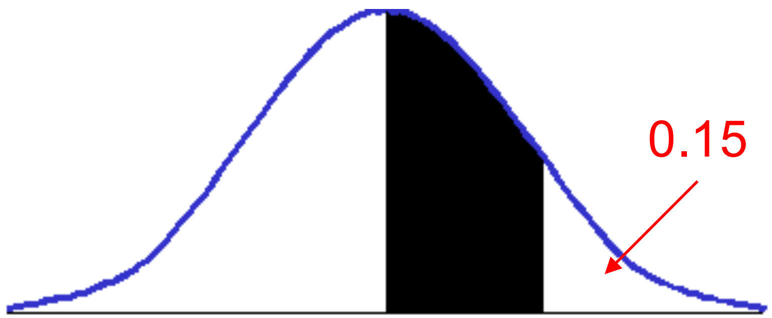
HW# 3 （給機率，反求X）

- X is normally distributed with mean 300 and standard deviation 40. What value of X does only the top 15 % exceed?

Step 1: 反求 P(X>0.15)的 Z

Step 2: 要查表0.35
- 因為0.5-0.15=0.35
- 可以求出Z＝

Step 3:帶入正規化公式，求X



Z=?

X=?

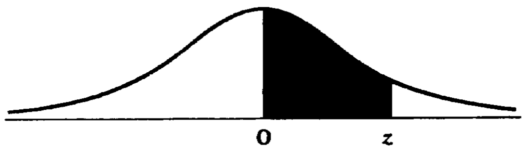
$$z_{.15} = \frac{x-\mu}{\sigma}$$

$$1.04 = \frac{x-300}{40}$$

$$\therefore x = 341.6$$

Normal Table **Areas of the Standard Normal Distribution**

The entries in this table are the probabilities that a random variable with a standard normal distribution assumes a value between 0 and z; the probability is represented by the shaded area under the curve in the accompanying figure. Areas for negative values of z are obtained by symmetry.



Second Decimal Place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
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0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
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0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
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1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
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2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999									
4.0	0.49997									

HW# 3

- X is normally distributed with mean 300 and standard deviation 40. What value of X does only the top 15 % exceed?

Step 1: 反求 P(X>0.15)的 Z

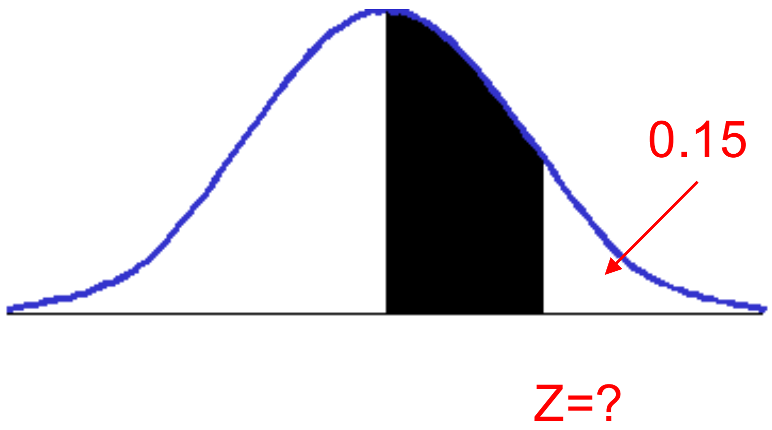
Step 2: 要查表0.35
- 因為0.5-0.15=0.35
- 可以求出Z＝

Step 3:帶入正規化公式，求X

$$z_{.15} = \frac{x - \mu}{\sigma}$$

$$1.04 = \frac{x - 300}{40}$$

$$\therefore x = 341.6$$



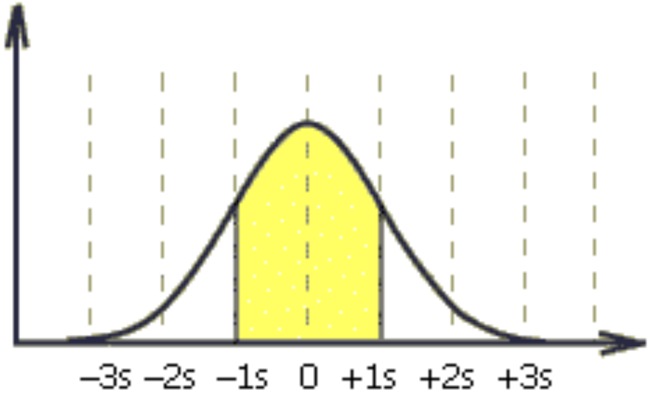
給機率，求X值（ＥＸＣＥＬ）
NORM.INV(p, u, sd)

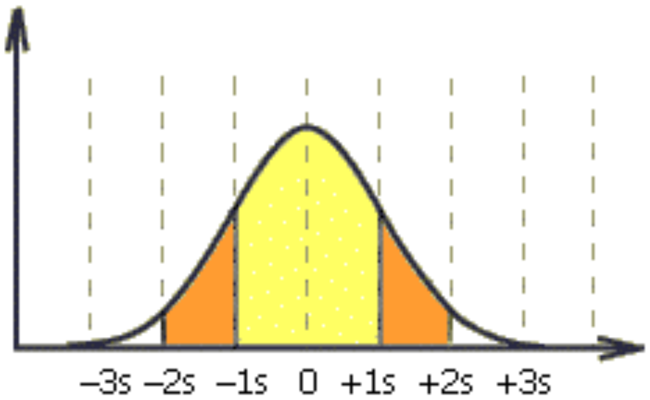
NORM.INV(0.85,300,40)
=341.6
＊EXCEL是給往下累積的機率

一、The Empirical Rule(常態分配)

Why 95% CI, 落在2倍標準差？（第4章）

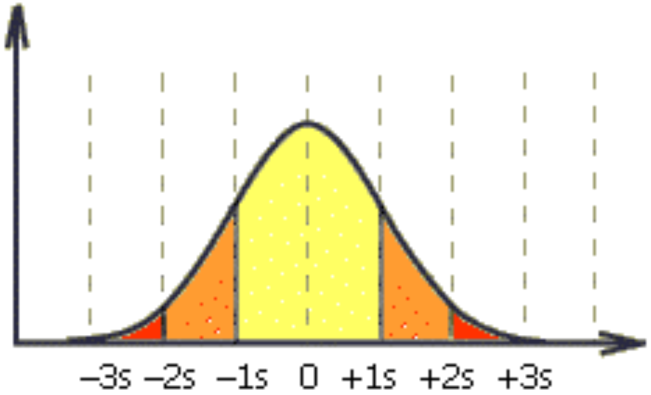
Approximately 68% of all observations fall within **one** standard deviation of the mean.





Approximately 95% of all observations fall within **two** standard deviations of the mean.

Approximately 99.7% of all observations fall within **three** standard deviations of the mean.

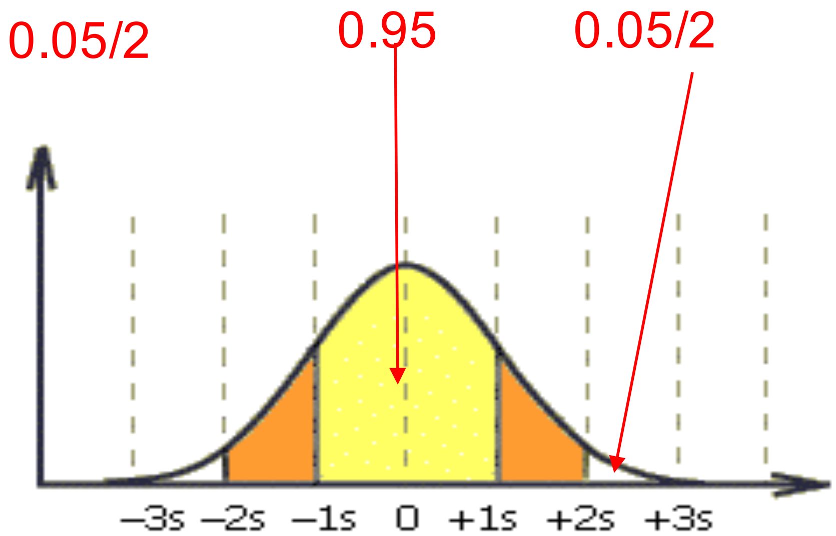


Why 95% CI, 落在2倍標準差？（第4章）

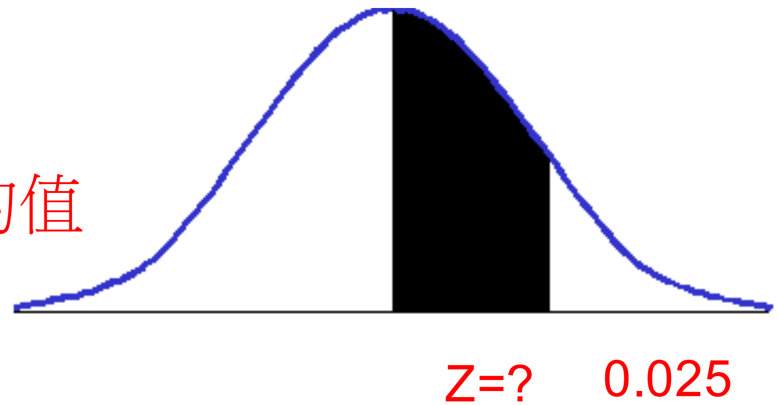
1. 0.05/2=0.025

2.表要查0.5-0.025=0.475

3. 可以求得 Z ＝1.96 (省略為2)

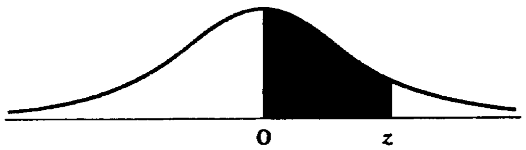


原來2倍標準差的2,是 Z score的值



Normal Table **Areas of the Standard Normal Distribution**

The entries in this table are the probabilities that a random variable with a standard normal distribution assumes a value between 0 and z; the probability is represented by the shaded area under the curve in the accompanying figure. Areas for negative values of z are obtained by symmetry.



Second Decimal Place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999									
4.0	0.49997									

HW#6

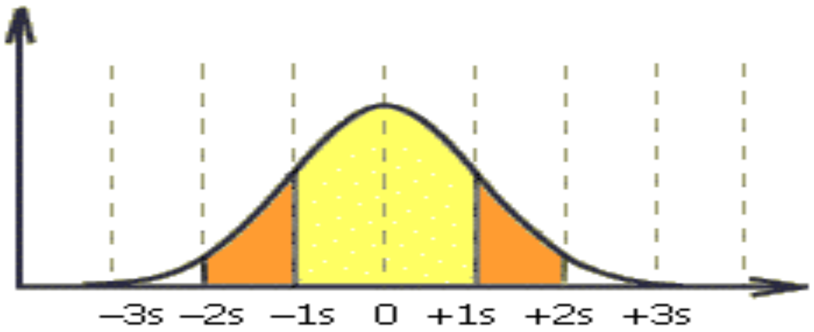
- 6. Consumers spend an average of $21 per week in cash without being aware of where it goes (data extracted from "Snapshots: A Hole in Our Pockets," USA Today, January 18, 2010, p.1A). Assume that the amount of cash spent without being aware of where it goes is normally distributed and that the standard deviation is $5.

a) What is the probability that a randomly selected person will spend more than $25?

b) What is the probability that a randomly selected person will spend between $10 and $20?

c) Between what two values will the middle 95% of the amounts of cash spent fall?

N(μ, σ)=N(21, 5)



Z=-1.96   Z=1.96

二、了解不同分配，如何運算
Exponential Distribution (指數分配)

- (兩個事件發生所需的時間)
- The exponential distribution can be used to model
  - the length of time between telephone calls
  - the length of time between arrivals at a service station
  - the life-time of electronic components.
- When the number of occurrences of an event follows the Poisson distribution, the time between occurrences follows the exponential distribution.

4. Exponential Distribution

- A random variable is exponentially distributed if its probability density function is given by

$$f(x) = \lambda e^{-\lambda x}, x \geq 0$$

- $E(X) = 1/\lambda$; $V(X) = (1/\lambda)^2$
- Finding exponential probabilities is relatively easy:
  - $P(X > a) = e^{-\lambda a}$.
  - $P(X < a) = 1 - e^{-\lambda a}$
  - $P(a_1 < X < a_2) = e^{-\lambda(a1)} - e^{-\lambda(a2)}$

4. Exponential Distribution

- Example (類似4,5,7,8)
  - The service rate at a movie theater checkout is 6 customers per hour.
  - If the service time is exponential, find the following probabilities:
    - A service is completed in 5 minutes,
    - A customer leaves the counter more than 10 minutes after arriving
    - A service is completed between 5 and 8 minutes.

Step1 :求出 lambda
Step2 :確定題目 X
Step3: 代入公式

4. Exponential Distribution

- Example (類似4,5,7,8)
  - The service rate at a movie theater checkout is 6 customers per hour.
  - If the service time is exponential, find the following probabilities:
    - A service is completed in 5 minutes,
    - A customer leaves the counter more than 10 minutes after arriving
    - A service is completed between 5 and 8 minutes.

Step1 :求出 u (or lambda)
Step2 :確定題目 X
Step3: 代入公式

*60分鐘內，會有6個人來
*10分鐘內，會有一個人來

*u=10分鐘（下一位客人來的時間）
E(X) = 1/λ, 10= 1/λ
(λ = 0.1).

4. Exponential Distribution

- A service is completed in 5 minutes,
- A customer leaves the counter more than 10 minutes after arriving
- A service is completed between 5 and 8 minutes.

- Solution
  - A service rate of 6 per hour = A service rate of .1 per minute (λ = .1/minute).
  - $P(X < 5) = 1-e^{-\lambda x} = 1 - e^{-.1(5)} = .3935$
  - $P(X > 10) = e^{-\lambda x} = e^{-.1(10)} = .3679$
  - $P(5 < X < 8) = e^{-.1(5)} - e^{-.1(8)} = .1572$

4. Exponential Distribution

- Solution
  - A service rate of 6 per hour = A service rate of .1 per minute (λ = .1/minute).
  - $P(X < 5) = 1\text{-}e^{-\lambda x} = 1 - e^{-.1(5)} = .3935$
  - $P(X > 10) = e^{-\lambda x} = e^{-.1(10)} = .3679$
  - $P(5 < X < 8) = e^{-.1(5)} - e^{-.1(8)} = .1572$

EXCEL
=EXPON.DIST(x, lambda, 1)

a. P(X < 5) =EXPON.DIST(5, 0.1, 1)=0.3934

b. P(X >10)= 1-EXPON.DIST(10, 0.1, 1)=0.3679

c. P(5 < X < 8)= EXPON.DIST(8, 0.1, 1)- EXPON.DIST(5, 0.1, 1)=0.1572

8. Suppose that the amount of time one spends in a bank is exponentially distributed with mean 10 minutes

a. What is the probability that a customer will spend more than 15 minutes in the bank?
b. What is the probability that a customer will spend more than 15 minutes in the bank given that he is still in the bank after 10 minutes?

Step1 :求出 u (or lambda)

*u=10分鐘（下一位客人來的時間）
$E(X) = 1/\lambda$, $10 = 1/\lambda$
($\lambda = 0.1$)

Step2 :確定題目 X
a. $P(X>15)$
b. $P(X>15 | X>10) = P(X>5)$

Step3: 代入公式

Memorylessness

An exponentially distributed random variable *T* obeys the relation

$$\Pr(T > s + t | T > s) = \Pr(T > t), \qquad \forall s, t \geq 0 .$$

When *T* is interpreted as the waiting time for an event to occur relative to some initial time, this relation implies that, if *T* is conditioned on a failure to observe the event over some initial period of time *s*, the distribution of the remaining waiting time is the same as the original unconditional distribution. For example, if an event has not occurred after 30 seconds, the conditional probability that occurrence will take at least 10 more seconds is equal to the unconditional probability of observing the event more than 10 seconds relative to the initial time.

The exponential distribution and the geometric distribution are the only memoryless probability distributions.

The exponential distribution is consequently also necessarily the only continuous probability distribution that has a constant Failure rate.

Poisson vs Exponential Distribution

- 作業#5用到的概念
- ＥＸ：一小時有6個客人(10鐘有1個人）

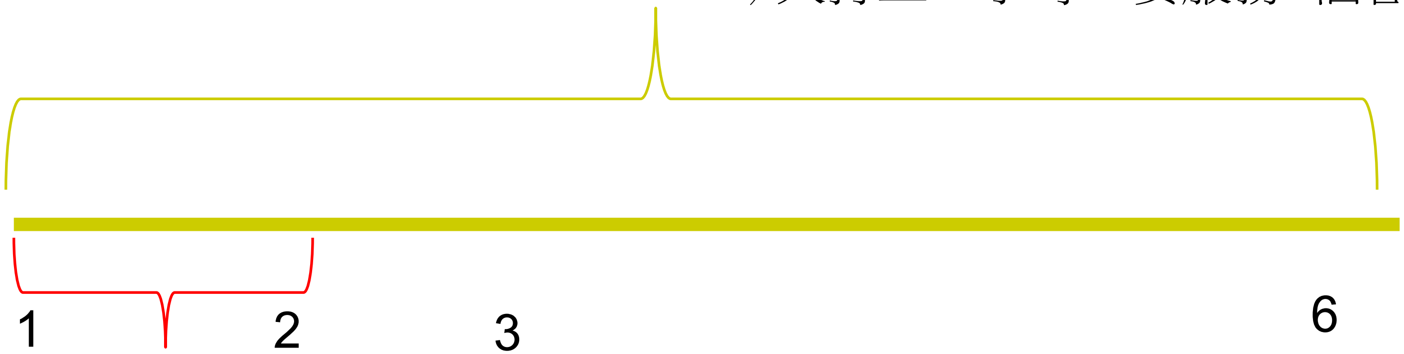
(a) $u_{Poisson}$ =

- 6 person（單位時間的來客數）

(b) $u_{exponential}$ =

- 1/6 hr （客人與客人間來的時間）
- λ = 1/u= 6 (帶公式要用6)

今天打工一小時，要服務6個客人(Poisson)



大概1/6 hr (10分鐘）後，下一個客人會來。 (exponential)

應用：下一個客人10分鐘後會來，假設服務約15分鐘，
表示客人要等5分鐘。是否多安排人力？(要配合出現的機率）

Project說明

統計的奧妙：生活到處有統計

Rule

- 分組作業
  - 最多4位同學1組

- 時間，某天的…
  - 下午 14:00-16:00 或 傍晚 19:00-20:30
- 地點
  - 新生南路(校門口對面)的誠品書局

- 繳交日期
  - 2016年11月24日
- 評分標準：
  - 採用相對成績。呈現越好的組別，成績越高。
  - 心得或解讀越好的，成績也會越高。

Procedure 1:蒐集資料

Poisson	Exponential

• Step 1
 	
• 每2分為一間隔，紀錄進入誠品書局的人數。
• 結伴前來者，無論人數，都算1位
	• Step 5
	
	• 每10秒為一單位，紀錄在某客人進入書局後，到下1位客人所需的時間。(4捨5入)
	    • 第1位客人與第2位客人間隔1分鐘，則紀錄為 6 單位時間。
	    • 結伴前來者，無論人數，都算1位

Hint:用excel打下所有資料 Ex: 14:00-14:02, 4 14:02-14:04, 0	think by yourself

Procedure 2:求出相對機率與u

Poisson	Exponential
• Step 2 • 將所記錄的人數與相對機率繪製機率分配圖 • Step 3 • 從相對機率分配圖計算μ	• Step 6 • 將所記錄的單位時間與相對機率繪製機率分配圖 • Step 7 • 從相對機率分配圖計算μ

每兩分鐘客人數(x)	頻率	相對頻率 f(x)	x*f(x)
0	0	0	0
1	0	0	0
2	1	0.022222222	0.044444444
3	2	0.044444444	0.133333333
4	4	0.088888889	0.355555556
5	6	0.133333333	0.666666667
6	7	0.155555556	0.933333333
7	6	0.133333333	0.933333333
8	12	0.266666667	2.133333333
9	3	0.066666667	0.6
10	2	0.044444444	0.444444444
11	2	0.044444444	0.488888889
12	0	0	0
total	45	1	$\sum x * f(x) = \mu =$ **6.733333.....**

2

Procedure 2:用excel 求出預測的機率值

Poisson	Exponential
• Step 4 • 以Poisson機率分配，參數為μ，計算預測機率分配 • 比較step 2 (實際資料的機率）與 step 4 （用u估計出來的機率）	• Step 8 • 以Exponential機率分配，參數為$\lambda = \frac{1}{\mu}$，計算預測機率分配並繪製機率密度分配圖
Hint: 1. Step 4可以用 Poisson.Dist (X, u, 0) 畫出 2.可畫長條圖比較實際與估計出來的資料	Hint: 1.Exponential是連續分配 （可參考講義圖形） think by yourself

比較Poisson與Exponential關係

- Step 9: Exponential 機率密度分配為連續分配，因此可以將Exponential機率密度分配，積分得到不連續的機率分配。並與Step 6 所得的觀測機率分配圖 (discrete 機率分配)作比較。

- 提示：
- 1.可以利用excel：EXPON.DIST(X, 1/u, 1)求得累加機率，在反求單點機率值。再將此值與step 6或step 7的值，畫圖比較，你就會發現很有趣的現象了。