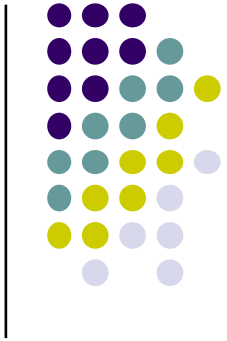
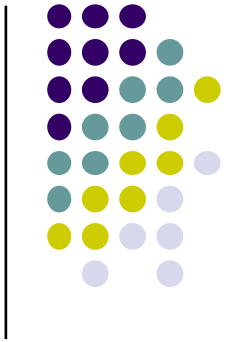


Ch 6 實習

Agenda



- 三種機率
  - ✓ Classical Approach...
  - ✓ Relative Frequency Approach
- 機率相關名詞與關係 （如何判斷題目）
  - ✓ Intersection, Union of Two Events ,Joint, Marginal
  - ✓ Conditional Probability
- 運算法則
  - ✓ Addition Rule
  - ✓ The Multiplication Rule
- 貝氏定理
  - ✓ Bayes' Law
  - ✓ Probability Trees
- 題目練習

一、 三種機率 (Relative Frequency Approach)

- 假設某廠商的訂貨如下列資料，請問他如果訂8 台，約略可以滿足多少的顧客滿意？
- （類似第8 題）

兩天demand	兩天# of days
0	5
1	16
2	13
3	12
4	23
5	33
6	23
7	34
8	10

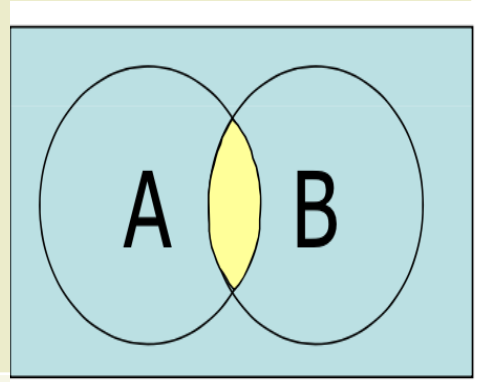
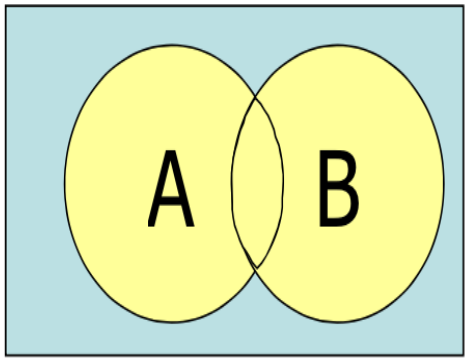
一、 三種機率 (Relative Frequency Approach)

- 假設某廠商的過去歷年訂貨如下列資料，請問他如果訂6台，約略可以滿足多少的顧客滿意？
- （類似第8 題）

兩天demand	兩天# of days	相對機率	累加機率
0	5	0.029585799	0.029585799
1	16	0.094674556	0.124260355
2	13	0.076923077	0.201183432
3	12	0.071005917	0.272189349
4	23	0.136094675	0.408284024
5	33	0.195266272	0.603550296
6	23	0.136094675	0.73964497
7	34	0.201183432	0.940828402
8	10	0.059171598	1
	169	1	

Step1:算出相對次數與累積相對次數
Step2:找到6 台可以滿足的機率, 73%

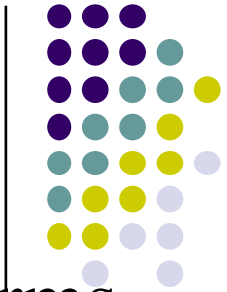
二、機率相關名詞與關係 （如何判斷題目）

語法	翻譯	關鍵字/ 公式	圖形意涵
$P(A \cap B)$ 交集	P(A and B)	Both, intersection, joint P(A and B) =P(A) *P(B) P(A and B)=P(A|B)*P(B) P(A and B)= P(B|A)*P(A) Multiplication Rule	
$P(A \cup B)$ 聯集	P (A or B)	Either, union P(A and B) =P(A) +P(B)-P (A and B) Addition Rule	
P$(A\|B)$ Conditional pro. (條件機率)		The probability of A given B P(A|B)=$\frac{P(A\ and\ B)}{P(B)}$	

- In a large city, two newspapers are published, the Sun and the Post. The circulation departments report that 22% of the city's households have a subscription to the Sun and 35% subscribe to the Post. A survey reveals that 6% of all households subscribe to both newspapers. What proportion of the city's households subscribe to either newspaper?
- P(A or B)

- What proportion of the city's households subscribe to both newspaper?
- P(A and B)

二、機率相關名詞與關係 （如何判斷題目）



- Determine the probability that a fund outperforms the market ***or*** the manager graduated from a top-20 MBA program.
- P(A or B)

- What's the probability that a fund will outperform the market ***given*** that the manager graduated from a top-20 MBA program?
- P(A|B)= P(outperform the market| graduated from a top-20 MBA program)

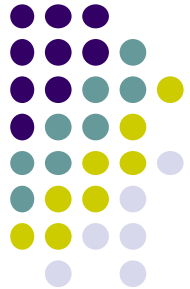
三、運算法則

The Multiplication Rule	
P(A and B) =P(A) *P(B)	AB are independent
P(A and B)=P(B)*P(A|B) P(A and B)= P(A)*P(B|A)	AB are not independent
Addition Rule	
P(A and B) =P(A) +P(B)-P (A and B)	
Conditional probability	
$P(A\|B)=\frac{P(A \text{ and } B)}{P(B)}$	

二、機率相關名詞與關係 （如何判斷題目）

- 請計算下列機率（類似題目7)
- P(A)=0.5, P(B)=0.1, 且ＡＢ獨立
- P(A∩B)?
  - ✓ =P(A)*P(B)=0.5*0.1=0.05
- P (A∪B) ?
  - ✓ = P(A)＋ P(B)- P(A∩B)= 0.5+0.1-0.05=0.55
- P(A|B) ?
  - ✓ = P(A∩B)/ P(B)= 0.05/0.1= 0.5
- P(A∩B^c)?
- =P(A)*P(B ^{c})=0.5*0.9=0.45

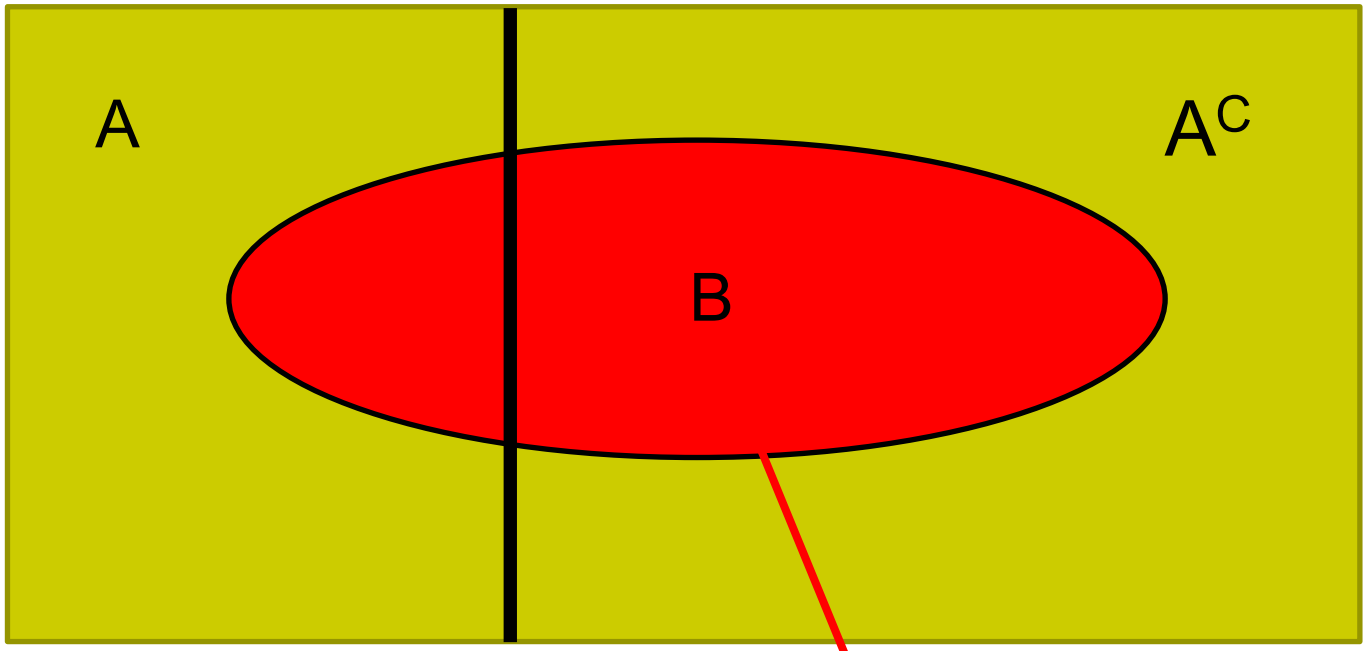
三、貝氏定理 (概念)



- The Graduate Management Admission Test (GMAT) is a requirement for all applicants of MBA programs. There are a variety of preparatory courses designed to help improve GMAT scores, which range from 200 to 800. Suppose that a survey of MBA students reveals that among GMAT scorers above 650, 52% took a preparatory course, whereas among GMAT scorers of less than 650 only 23% took a preparatory course. An applicant to an MBA program has determined that he needs a score of more than 650 to get into a certain MBA program, but he feels that his probability of getting that high a score is quite low--10%. He is considering taking a preparatory course that cost $500. He is willing to do so only if his probability of achieving 650 or more doubles. What should he do?
- 現在有幾個事件？
- (A: ＧＭＡＴ>650, B:去補習)

三、貝氏定理 (概念)

A：GMAT > 650 (prior)　　A^c :GMAT <650 (prior)

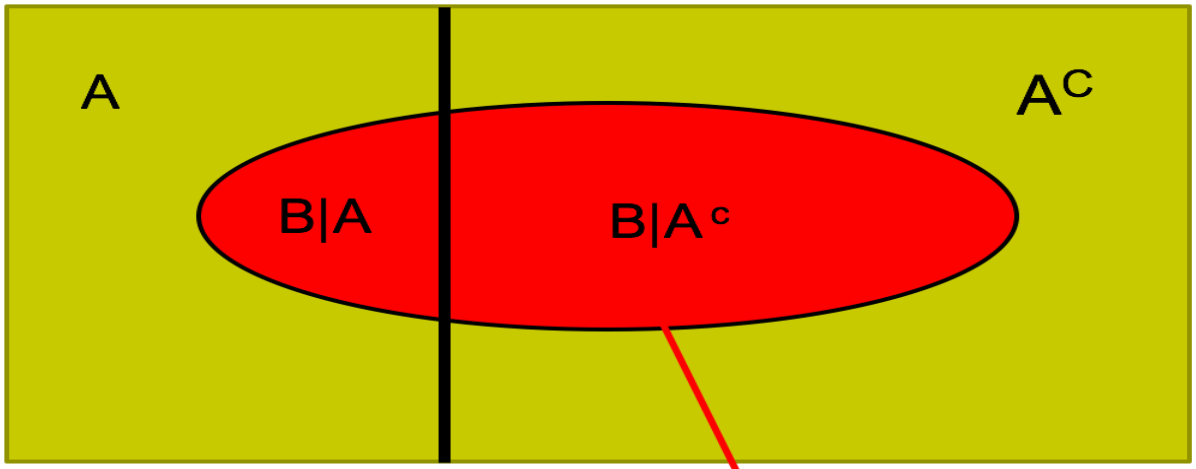


B: 有參加補習班 (after)

已知： P（B|A）=P (有補習 | 多益 > 650)
反求： P（A| B) = P (多益> 650 |去補習）

三、貝氏定理 (概念)

A：GMAT> 650 (prior)　A^c GMAT <650 (prior)



B: 有參加補習班 (after)

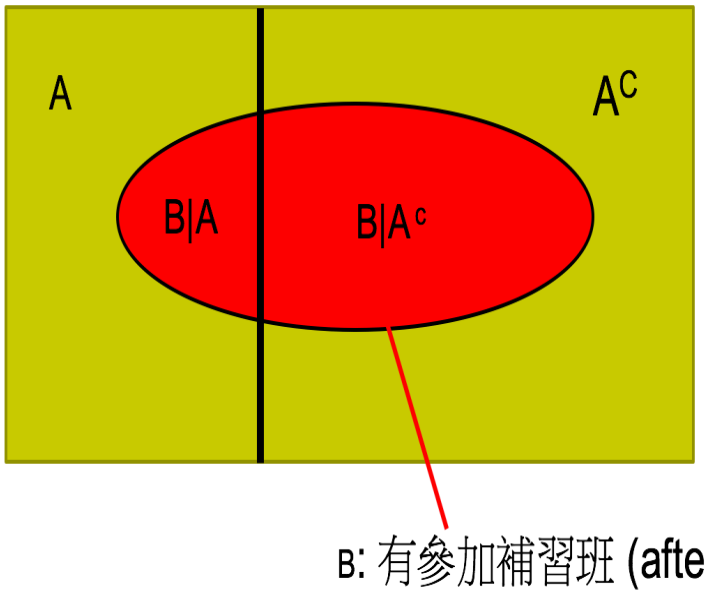
$$P(A|B)=\frac{P(A\ and\ B)}{P(B)}=\frac{P(A\ and\ B)}{P(B\ and\ A)+P(B\ and\ A^c)}=\frac{P(A)*P(B|A)}{P(A)*P(B|A)+P(A)*P(B|A^c)}$$

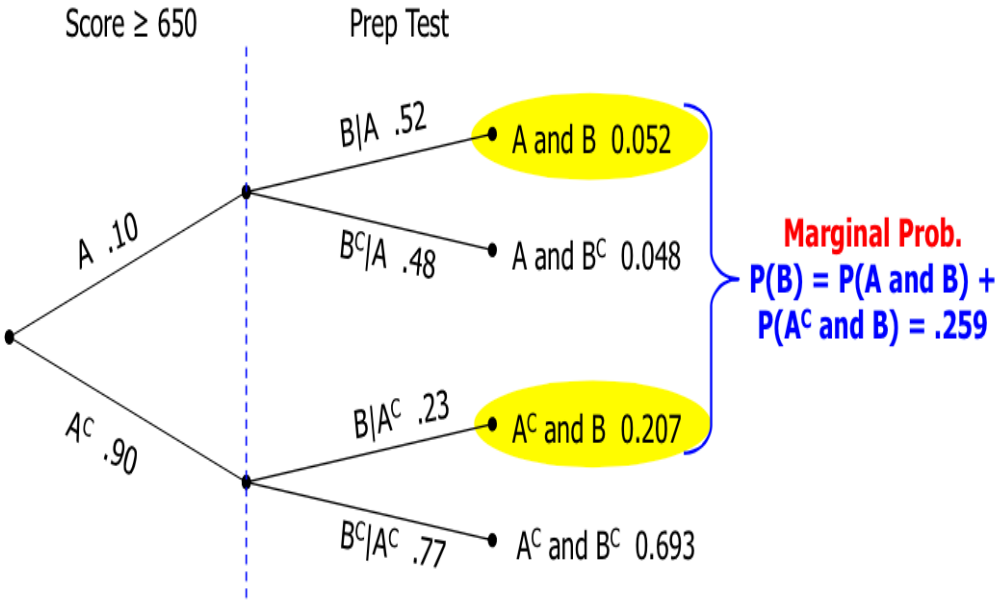
P(A and B)=P(B)*P(A|B)……1
P(A and B)= P(A)*P(B|A)……2
為何用2不用1?

因為P(A)事前機率（你已經知道的機率）

三、貝氏定理 (概念)

A：GMAT> 650 (prior)　A^c GMAT <650 (prior)





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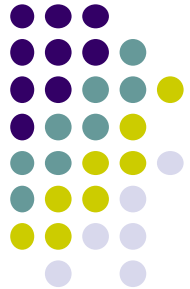
P(A and B)=P(B)*P(A|B)……1
P(A and B)= P(A)*P(B|A)……2,

為何用2不用1？ 因為P(A)事前機率（你已經知道的機率）

三、貝氏定理 (概念)



- The Graduate Management Admission Test (GMAT) is a requirement for all applicants of MBA programs. There are a variety of preparatory courses designed to help improve GMAT scores, which range from 200 to 800. Suppose that a survey of MBA students reveals that among GMAT scorers above 650, 52% took a preparatory course, whereas among GMAT scorers of less than 650 only 23% took a preparatory course. An applicant to an MBA program has determined that he needs a score of more than 650 to get into a certain MBA program, but he feels that his probability of getting that high a score is quite low--10%. He is considering taking a preparatory course that cost $500. He is willing to do so only if his probability of achieving 650 or more doubles. What should he do?

三、貝氏定理 (類似題目3,5,6)

- A: GMAT >650
- A^c : GMAT <650
- B: 去補習
- 已知 P(A)=0.1, $P(A^c)$=0.9
- P(B|A)=0.52, $P(B|A^c)$=0.23
- 求P(A|B)？

(1) 0.201 這是什麼意思？

該學生參加補習，
他的ＧＭＡＴ有20%機率會高於650

(2) 所以該學生，應該去補習嗎？

該學生應該參加補習，
因為他原本考到650機率為 0.1,
他說機率翻倍，他就補習。

$$P(A|B)=\frac{P(A\ and\ B)}{P(B)}=\frac{P(A\ and\ B)}{P(B\ and\ A)+P(B\ and\ A^c)}=\frac{P(A)*P(B|A)}{P(A)*P(B|A)+P(A^c)*P(B|A^c)}$$

$$P(A|B)=\frac{0.1*0.52}{0.1*0.52+0.9*0.23}=\frac{0.052}{0.259}=0.201$$

三、貝氏定理 (類似題目3,5,6)

3. To help select suitable employees for a particular job a personnel department administers and aptitude test to all applicants. To test the effectiveness of the test a sample of applicants who failed were also hired and given a fast trial at the job. It was found that of the 30 percent who passed the test, 80% were satisfactory, and of those who did not, only 10 percent were satisfactory

a. What is the probability that an applicant selected at random will prove to be satisfactory at this job?

b. If an applicant is satisfactory, what is the probability that he passed the test?

Step 1: 定義事件

A: 有通過測試(prior) , A^c :沒有通過測驗 (prior)

B:後來工作滿意 (After)

Step 2:確定題目，問什麼？

a. P(後來滿意工作）＝P(B)？

b. P(通過考試｜後來工作滿意）＝P(A|B)

Step 3:代公式

三、貝氏定理 (類似題目3,5,6)

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Step 1: 定義事件

A: 有通過測試 , A^c :沒有通過測驗

B:後來工作表現佳

Step 2:確定題目，問什麼？

a. P(後來滿意工作）＝P(B)？

b. P(通過考試｜後來工作滿意）＝P(A|B)

Step 3:代公式

$$P(A|B)=\frac{P(A\ and\ B)}{P(B)}=\frac{P(A\ and\ B)}{P(B\ and\ A)+P(B\ and\ A^c)}=\frac{P(A)*P(B|A)}{P(A)*P(B|A)+P(A^c)*P(B|A^c)}$$

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a. What is the probability that an applicant selected at random will prove to be satisfactory at this job?

b. If an applicant is satisfactory, what is the probability that he passed the test?

Step 1: 定義事件

A: 有通過測試 , P(A)=0.3

A^c :沒有通過測驗, $P(A^c)$=0.7

B:後來工作表現佳 P(B)= ?

P(B|A)=0.8

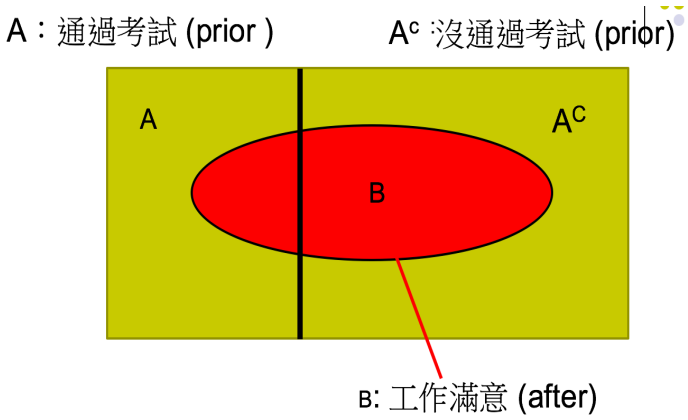
$P(B|A^c)$=0.1

Step 2:確定題目，問什麼？

a. P(後來滿意工作）＝P (B)？

b. P(通過考試｜後來工作滿意）=P(A|B)

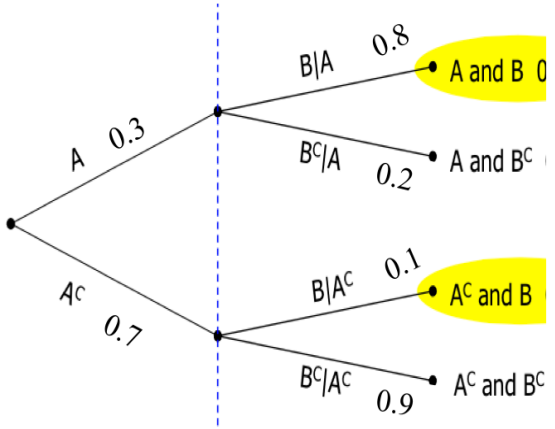
Step 3:代公式



a. $P(B)= P(A) * P(B|A) + P(A^c) * P(B|A^c) = 0.3 * 0.8 + 0.7 * 0.1 = 0.31$

b. $P(A|B)= \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \text{ and } B)}{P(B \text{ and } A)+P(B \text{ and } A^c)} = \frac{P(A)*P(B|A)}{P(A)*P(B|A)+P(A^c)*P(B|A^c)}$

$P(A|B)= \frac{0.3*0.8}{0.3*0.8+0.7*0.1} = 0.7742$



表示問卷準確度蠻高的

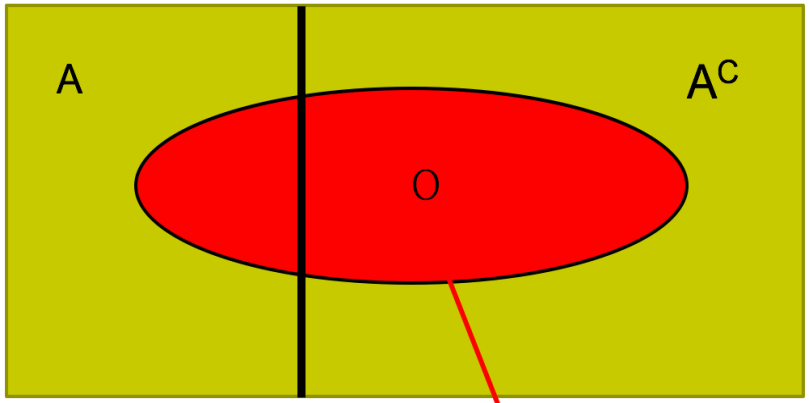
4. 研究者若善用統計技巧，將可降低受訪者心防，使調查結果更加可靠。例如某位老師想了解班上 60 名同學中，約有多少比率在國中前談過戀愛，於是他設計了A、B 兩份問卷(其中 A 卷 40 張，B 卷 20 張)及不記名答案卷 60 張，隨機發給班上同學(同學不是拿到 A 卷就是 B 卷，至於 A、B 卷接不回收，答案卷則要回收)。其中 A 卷的題目是我曾經在國中以前談過戀愛，因此拿到 A 卷的同學若國中前曾談戀愛，就要在答案卷上劃 O，若不曾談過戀愛，則劃 X；反之，B 卷的題目是我不曾在國中以前談過戀愛，因此拿到 B 卷的同學若國中前曾談戀愛，就要在答案卷上劃 X，若不曾談過戀愛，則劃 O。如此設計，不論某位同學回答的結果是 O 或 X，老師都無法得知該生在國中前是否談過戀愛，但卻可大致了解全班的狀況。答案卷經回收後，老師發現 60 張答案卷中有 25 張劃 O，35 張劃 X。請你求出在本班同學中，約有多少比例的同學曾在國中前談過戀愛?

A：拿A卷
A^c：拿B卷
B：畫O事件
X:表示有談過戀愛

目標：找到P（X）

提示：
1. 先畫出機率樹
2. 求P(O)公式,
3. 再求P(X)

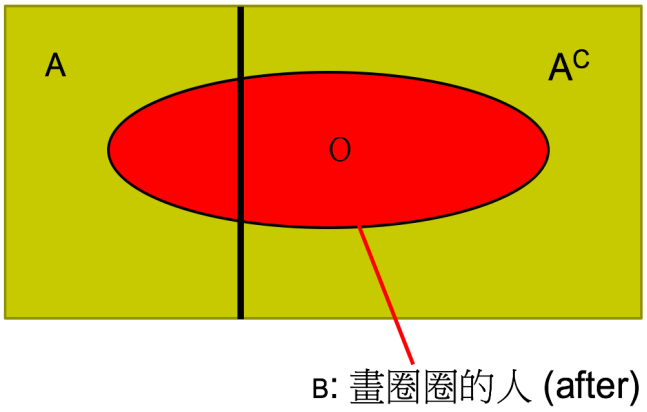
A：拿A卷　　A^c ：拿B卷



B: 畫圈圈的人 (after)

4. 研究者若善用統計技巧，將可降低受訪者心防，使調查結果更加可靠。例如某位老師想了解班上 60 名同學中，約有多少比率在國中前談過戀愛，於是他設計了 A、B 兩份問卷(其中 A 卷 40 張，B 卷 20 張)及不記名答案卷 60 張，隨機發給班上同學(同學不是拿到 A 卷就是 B 卷，至於 A、B 卷接不回收，答案卷則要回收)。其中 A 卷的題目是我曾經在國中以前談過戀愛，因此拿到 A 卷的同學若國中前曾談戀愛，就要在答案卷上劃 O，若不曾談過戀愛，則劃 X；反之，B 卷的題目是我不曾在國中以前談過戀愛，因此拿到 B 卷的同學若國中前曾談戀愛，就要在答案卷上劃 X，若不曾談過戀愛，則劃 O。如此設計，不論某位同學回答的結果是 O 或 X，老師都無法得知該生在國中前是否談過戀愛，但卻可大致了解全班的狀況。答案卷經回收後，老師發現 60 張答案卷中有 25 張劃 O，35 張劃 X。請你求出在本班同學中，約有多少比例的同學曾在國中前談過戀愛?

A：拿A卷　　A^c ：拿B卷



a. P(B)=

$P(B) = P(A) * P(B|A) + P(A^c) * P(B|A^c)$

$\frac{25}{60} = \frac{40}{60} * P(B|A) + \frac{20}{60} * P(B|A^c)$

$\frac{25}{60} = \frac{40}{60} * P(X) + \frac{20}{60} * P(B|A^c)$*(1-P(X))

* 假設 A B 要獨立事件（只能拿一種考卷）