Assignment 9 Solution

1.

(a) H_0: $\mu = 14.6$ hours
H_1: $\mu \neq 14.6$ hours

(b) A Type I error is the mistake of concluding that the mean number of hours studied at your school is different from the 14.6 hour benchmark reported by *Business Week* when in fact it is not any different.

(c) A Type II error is the mistake of not concluding that the mean number of hours studied at your school is different from the 14.6 hour benchmark reported by *Business Week* when it is in fact different.

2.

(a)

H_0: $\mu = 375$. The mean life of a large shipment of light bulbs is equal to 375 hours.
H_1: $\mu \neq 375$. The mean life of a large shipment of light bulbs differs from 375 hours.
Decision rule: Reject H_0 if $|Z_{STAT}| > 1.96$

Test statistic: $Z_{STAT} = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \dfrac{350 - 375}{100/\sqrt{64}} = -2$

Decision: Since $|Z_{STAT}| > 1.96$, reject H_0. There is enough evidence to conclude that the mean life of a large shipment of light bulbs differs from 375 hours.

(b) p-value = 0.0455. If the population mean life of a large shipment of light bulbs is indeed equal to 375 hours, the probability of obtaining a test statistic that is more than 2 units away from 0 is 0.0455.

(c) $\bar{X} \pm Z_{a/2} \dfrac{\sigma}{\sqrt{n}} = 350 \pm 1.96 \dfrac{100}{\sqrt{64}}$ $325.5005 \leq \mu \leq 374.4995$

(d) You are 95% confident that the population mean life of a large shipment of light bulbs is somewhere between 325.5005 and 374.4995 hours.
Since the 95% confidence interval does not contain the hypothesized value of 375, you will reject H_0. The conclusions are the same.

3.

(a) H_0: $\mu = 50$ vs. H_1: $\mu \neq 50$

(b) Test statistic: $z = -1.40$

Rejection region: $| z | > z_{.025} = 1.96$

Conclusion: Don't reject H_0. We cannot infer that the process is out of proper adjustment.

(c) LCL = 46.64, and UCL = 50.56. Since the hypothesized value 50 falls in the 95% confidence interval, we fail to reject H_0 at $\alpha = 0.05$.

4.

(a) $\beta = P(884.2 < \bar{x} < 1015.8$ given that $\mu = 1000) = P(-2.9 < z < .40) = .6535$

(b) Power = $1 - \beta = 1 - 0.6535 = 0.3465$

(c) The probability of detecting that the mean lifetime is not 950 hours, when indeed the lifetime is 1,000 hours, is 0.3465, when $\alpha = 0.10$.

(d) $\beta = P(897.98 < \bar{x} < 1002.02$, given that $\mu = 1000) = P(-3.23 < z < .06) = .5233$

(e) β decreases as n increases.

5.

i. Rejection region： $\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_\alpha$

$$\frac{\bar{x}-10}{3/\sqrt{100}} < -z_{.01} = -2.33$$

$\bar{x} < 9.30$

$$\beta = P(\bar{x} > 9.30 \text{ given } \mu = 9) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{9.30-9}{3/\sqrt{100}}\right) = P(z > 1) = 1 - .8413 = .1587$$

ii Rejection region: $\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_\alpha$

$$\frac{\bar{x}-10}{3/\sqrt{75}} < -z_{.05} = -1.645$$

$\bar{x} < 9.43$

$$\beta = P(\bar{x} > 9.43 \text{ given } \mu = 9) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{9.43-9}{3/\sqrt{75}}\right) = P(z > 1.24) = 1 - .8925 = .1075$$

iii Rejection region: $\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_\alpha$

$$\frac{\bar{x}-10}{3/\sqrt{50}} < -z_{.10} = -1.28$$

$\bar{x} < 9.46$

$$\beta = P(\bar{x} > 9.46 \text{ given } \mu = 9) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{9.46-9}{3/\sqrt{50}}\right) = P(z > 1.08) = 1 - .8599 = .1401$$

Plan ii has the lowest probability of a type II error.

6.

z-Estimate: Mean

	Cost
Mean	1810.156
Standard Deviation	323.4462
Observations	64
SIGMA	400
LCL	1712.158
UCL	1908.154

7.

$H_0 : \mu = 560$

$H_1 : \mu > 560$

Z-Test: Mean

	GMAT
Mean	569
Standard Deviation	41.8745
Observations	20
Hypothesized Mean	560
SIGMA	50
z Stat	0.805
P(Z<=z) one-tail	0.2104
z Critical one-tail	1.6449
P(Z<=z) two-tail	0.4208
z Critical two-tail	1.96

z=0.805

P-value=P(Z>0.805)=0.2104

There is enough evidence to conclude that the dean's claim is true.

8.

$H_0 : \mu = 4$

$H_1 : \mu \neq 4$

Z-Test: Mean

	Ski Days
Mean	4.9048
Standard Deviation	2.0769
Observations	63
Hypothesized Mean	4
SIGMA	2
z Stat	3.5907
P(Z<=z) one-tail	0.0002
z Critical one-tail	1.2816
P(Z<=z) two-tail	0.0004
z Critical two-tail	1.6449

z=3.5907

P-value=2P(Z>3.5907)=0.0004

There is enough evidence to infer that the average Alpine skier does not ski 4 times per year.