Assignment 6 Solution

1.

(a) $P(X < 95) = \text{P}(Z < -2.50) = 0.0062$

(b) $P(95 < \overline{X} < 97.5) = P(-2.50 < Z < -1.25) = 0.1056 - 0.0062 = 0.0994$

(c) $P(\overline{X} > 102.2) = P(Z > 1.10) = 1.0 - 0.8643 = 0.1357$

(d) $P(\overline{X} > A) = P(Z > -0.39) = 0.65$ $\quad \overline{X} = 100 - 0.39(\frac{10}{\sqrt{25}}) = 99.22$

2.

(a) $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{25}} = 0.4$

PHStat output:

Probability for a Range	
From X Value	7.8
To X Value	8.2
Z Value for 7.8	-0.5
Z Value for 8.2	0.5
P(X<=7.8)	0.3085
P(X<=8.2)	0.6915
P(7.8<=X<=8.2)	0.3829

$P(7.8 < \overline{X} < 8.2) = P(-0.50 < Z < 0.50) = 0.6915 - 0.3085 = 0.3830$

(b) PHStat output:

Probability for a Range	
From X Value	7.5
To X Value	8
Z Value for 7.5	-1.25
Z Value for 8	0
P(X<=7.5)	0.1056
P(X<=8)	0.5000
P(7.5<=X<=8)	0.3944

$P(7.5 < \overline{X} < 8.0) = P(-1.25 < Z < 0) = 0.5 - 0.1056 = 0.3944$

(c) $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.2$

PHStat output:

Probability for a Range	
From X Value	7.8
To X Value	8.2
Z Value for 7.8	-1
Z Value for 8.2	1
P(X<=7.8)	0.1587
P(X<=8.2)	0.8413
P(7.8<=X<=8.2)	0.6827

$P(7.8 < \overline{X} < 8.2) = P(-1.00 < Z < 1.00) = 0.8413 - 0.1587 = 0.6826$

(d) With the sample size increasing from $n = 25$ to $n = 100$, more sample means will be closer to the distribution mean. The standard error of the sampling distribution of size 100 is much smaller than that of size 25, so the likelihood that the sample mean will fall within ±0.2 minutes of the mean is much higher for samples of size 100 (probability = 0.6826) than for samples of size 25 (probability = 0. 3830).

3.

(a) $P(1.99 < \overline{X} < 2.00) = P(-1.00 < Z < 0) = 0.5 - 0.1587 = 0.3413$

(b) $P(\overline{X} < 1.98) = P(Z < -2.00) = 0.0228$

(c) $P(\overline{X} > 2.01) = P(Z > 1.00) = 1.0 - 0.8413 = 0.1587$

(d) $P(\overline{X} > A) = P(Z > -2.33) = 0.99$ $\quad A = 2.00 - 2.33(0.01) = 1.9767$

(e) $P(A < X < B) = P(-2.58 < Z < 2.58) = 0.99$

$A = 2.00 - 2.58(0.01) = 1.9742 \quad B = 2.00 + 2.58(0.01) = 2.0258$

4.

(a) $\mu_p = \pi = 0.501$, $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.501(1-0.501)}{100}} = 0.05$

Partial PHstat output:

Probability for X >	
X Value	0.55
Z Value	0.98
P(X>0.55)	0.1635

$P(p > 0.55) = P(Z > 0.98) = 1 - 0.8365 = 0.1635$

(b) $\mu_p = \pi = 0.60$, $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.6(1-0.6)}{100}} = 0.04899$

Partial PHstat output:

Probability for X >	
X Value	0.55
Z Value	-1.020621
P(X>0.55)	0.8463

$P(p > 0.55) = P(Z > -1.021) = 1 - 0.1539 = 0.8461$

(c) $\mu_p = \pi = 0.49$, $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.49(1-0.49)}{100}} = 0.05$

Partial PHstat output:

Probability for X >	
X Value	0.55
Z Value	1.2002401
P(X>0.55)	0.1150

$P(p > 0.55) = P(Z > 1.20) = 1 - 0.8849 = 0.1151$

5.

(a) PHStat output:

Probability for a Range	
From X Value	**0.3**
To X Value	**0.4**
Z Value for 0.3	**-1.767767**
Z Value for 0.4	**1.178511**
P(X<=0.3)	**0.0385**
P(X<=0.4)	**0.8807**
P(0.3<=X<=0.4)	**0.8422**

Since $n = 200$, which is quite large, we use the sample proportion to approximate the population proportion and, hence, $\pi = 0.36$. Also the sampling distribution of the sample proportion will be close to a normal distribution according to the central limit theorem.

$\mu_p = \pi = 0.36$, $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.36(1-0.36)}{200}} = 0.0339$

$P(0.3 < p < 0.4) = P(-1.7678 < Z < 1.1785) = 0.8422$

(b)

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	5.00%
Z Value	-1.644854
X Value	0.304172

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	95.00%
Z Value	1.644854
X Value	0.415828

$P(A < p < B) = P(-1.6449 < Z < 1.6449) = 0.90$

$A = 0.3041$

$B = 0.4158$

The probability is 90% that the sample percentage will be contained within 5.5828% symmetrically around the population percentage.

(c) PHStat output:

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	2.50%
Z Value	-1.959964
X Value	0.293477

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	97.50%
Z Value	1.959964
X Value	0.426523

$P(A < p < B) = P(-1.96 < Z < 1.96) = 0.95$

$A = 0.2935$

$B = 0.4265$

The probability is 95% that the sample percentage will be contained within 6.6523% symmetrically around the population percentage.

6.

(a) $\mu_p = \pi = \frac{1}{25} = .04$, $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.04(1-0.04)}{100}} = 0.0196$

PHStat output:

Probability for a Range	
From X Value	0.05
To X Value	0.1
Z Value for 0.05	0.51031
Z Value for 0.1	3.061862
P(X<=0.05)	0.6951
P(X<=0.1)	0.9989
P(0.05<=X<=0.1)	0.3038

$P(0.05 < p < 0.10) = P\ (..5103 < Z < 3.0619) = 0.3038$

(b) PHStat output:

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	5.00%
Z Value	-1.644854
X Value	0.007768

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	95.00%
Z Value	1.644854
X Value	0.072232

$P(A < p < B) = P(-1.6449 < Z < 1.6449) = 0.90$
$A = 0.04 - 1.6449(0.0196) = 0.0078$
$B = 0.04 + 1.6449(0.0196) = 0.0722$
The probability is 90% that the sample percentage will be contained within 3.223% symmetrically around the population percentage.

(c) Partial PHStat output:

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	2.50%
Z Value	-1.959964
X Value	0.001593
Find X and Z Given Cum. Pctage.	
Cumulative Percentage	97.50%
Z Value	1.959964
X Value	0.078407

$P(A < p < B) = P(-1.9600 < Z < 1.9600) = 0.95$
$A = 0.04 - 1.9600(0.0196) = 0.0016$
$B = 0.04 + 1.9600(0.0196) = 0.0784$
The probability is 95% that the sample percentage will be contained within 3.84% symmetrically around the population percentage.

7.

$$\boldsymbol{Pr}(\bar{\boldsymbol{X}}_1 - \bar{\boldsymbol{X}}_2 > \boldsymbol{0}) = \Pr\left(Z > \frac{0-10}{12.96}\right)$$
$$= \Pr(Z > -0.77) = 1 - \Pr(Z < -0.77)$$
$$= 1 - 0.22 = 0.78$$

- So there is a 78% probability that worker 1 will outperform worker 2 over a five day period.

8.

$$P(\bar{X}_1 - \bar{X}_2 > 0) = P\left(\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{0 - (40 - 38)}{\sqrt{\frac{6^2}{25} + \frac{8^2}{25}}}\right) = P(Z > -1.00) = 1 - P(Z < -1.00)$$

$1 - .1587 = .8413$