Assignment 4 Solution

1.

- a.
  - E(X) = 0(0.4) +1(0.3) + 2(0.2) +3(0.1) = 1
  - $V(X)=E(X^2)-\{E(X)\}^2$
    $=(0)^2(0.4)+(1)^2(0.3)+(2)^2(0.2)+(3)^2(0.1)-(1)^2 = 1$
  - $\sigma(X)=1$
- b.

x	0	1	2	3
y	2	5	8	11
P(y)	0.4	0.3	0.2	0.1

- c.
  - E(Y) = 2(0.4) + 5(0.3) + 8(0.2) + 11(0.1) = 5
  - $V(Y)=E(Y^2)-\{E(Y)\}^2$
    $=(2)^2(0.4)+(5)^2(0.3)+(8)^2(0.2)+(11)^2(0.1)-(5)^2 = 9$
  - $\sigma(Y)=3$
- d.
  - E(Y) = E(3X+2) = 3E(X)+2 = 5
  - V(Y) = V(3X+2) = 9V(X) = 9

2.

a

x	P(x)
1	.7
2	.2
3	.1

y	P(y)
1	.6
2	.4

b $\mu_x = E(X) = \sum xP(x) = 1(.7) + 2(.2) + 3(.1) = 1.4$

$\sigma^2 = V(X) = \sum (x-\mu)^2 P(x) = (1-1.4)^2(.7) + (2-1.4)^2(.2) + (3-1.4)^2(.1) = .44$

$\mu_y = E(Y) = \sum yP(y) = 1(.6) + 2(.4) = 1.4$

$\sigma^2 = V(Y) = \sum (y-\mu)^2 P(y) = (1-1.4)^2(.6) + (2-1.4)^2(.4) = .24$

$\sum_{\text{all x}} \sum_{\text{all y}} xyP(x, y) = (1)(1)(.42) + (1)(2)(.28) + (2)(1)(.12) + (2)(2)(.08) + (3)(1)(.06) + (3)(2)(.04) = 1.96$

$COV(X, Y) = \sum_{\text{all x}} \sum_{\text{all y}} xyP(x, y) - \mu_x\mu_y = 1.94 - (1.4)(1.4) = 0$

$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{.44} = .66,\ \sigma_y = \sqrt{\sigma_y^2} = \sqrt{.24} = .49$

$\rho = \dfrac{COV(X, Y)}{\sigma_x\sigma_y} = \dfrac{0}{(.66)(.49)} = 0$

c

x + y	P(x + y)
2	.42
3	.40
4	.14
5	.04

3.

(a) $E(P) = (0.4)(\$50) + (0.6)(\$100) = \$80$

(b) $\sigma_P = \sqrt{(.4)^2(9000) + (.6)^2(15000) + 2(.4)(.6)(7500)} = 102.18$

4.

Let X = number of on-time flights.

(a) $P(X = 4) = 0.2128.$

(b) $P(X = 6) = 0.3153$

(c) $P(X > 5) = 0.9294$

(d) $E(X) = 4.95 \qquad \sigma_X = 0.9307$

5.

(a) $P(X = 0) = 0.0834$

(b) $P(X = 1) = 0.2351$

(c) $P(X \leq 2) = 0.6169$

(d) $P(X \geq 3) = 0.3831$

6.

(a) mean = 1 standard deviation = 0.9747

(b) $P(X = 0) = 0.3585$ (c) $P(X = 1) = 0.3774$ (d) $P(X \geq 2) = 0.2642$

7.

(a) $P(X < 5) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$$= \frac{e^{-6}(6)^0}{0!} + \frac{e^{-6}(6)^1}{1!} + \frac{e^{-6}(6)^2}{2!} + \frac{e^{-6}(6)^3}{3!} + \frac{e^{-6}(6)^4}{4!}$$

$$= 0.002479 + 0.014873 + 0.044618 + 0.089235 + 0.133853 = 0.2851$$

(b) $P(X = 5) = \frac{e^{-6}(6)^5}{5!} = 0.1606$

(c) $P(X \geq 5) = 1 - P(X < 5) = 1 - 0.2851 = 0.7149$

(d) $P(X = 4 \text{ or } X = 5) = P(X = 4) + P(X = 5) = \frac{e^{-6}(6)^4}{4!} + \frac{e^{-6}(6)^5}{5!} = 0.2945$

8.

a $P(X = 5 \text{ with } \mu = 14/3) = \frac{e^{-\mu}\mu^x}{x!} = \frac{e^{-14/3}(14/3)^5}{5!} = .1734$

b. $P(X = 1 \text{ with } \mu = 1/3) = \frac{e^{-\mu}\mu^x}{x!} = \frac{e^{-1/3}(1/3)^1}{1!} = .2388$