Assignment 5 Solution

1

- f(x) = $\frac{1}{(175-110)} = \frac{1}{65}$,110 ≦ x ≦ 175
- a. P(X ≧ 150) = $(175-150)\frac{1}{65}$ = 0.3846
- b. P(120 ≦ X ≦ 160) = $(160-120)\frac{1}{65}$ = 0.6154

2.

- P(0 < Z < $z_{.15}$) = 1-0.15 = 0.85
- $z_{.15}$ = 1.04; $z_{.15} = \frac{x-\mu}{\sigma}$

$$1.04 = \frac{x-300}{40}$$

$$\therefore x = 341.6$$

3.

- a. P(5 < X < 10) = $P\left(\frac{5-7.2}{1.9} < \frac{X-\mu}{\sigma} < \frac{10-7.2}{1.9}\right)$
  = P(–1.16 < Z < 1.47) = 0.9292-(1-0.8770)
  = .8062
- b. P(X > 7) = $P\left(\frac{X-\mu}{\sigma} > \frac{7-7.2}{1.9}\right)$ = P(Z > –.11) = 0.5438
- c. P(X < 4) = $P\left(\frac{X-\mu}{\sigma} < \frac{4-7.2}{1.9} = -1.68\right)$
  =1-0.9535= .0465

4.

- A service rate of 6 per hour =
  A service rate of .1 per minute (λ = .1/minute).
- $P(X < 5) = 1-e^{-\lambda x} = 1 - e^{-.1(5)} = .3935$
- $P(X > 10) = e^{-\lambda x} = e^{-.1(10)} = .3679$
- $P(5 < X < 8) = e^{-.1(5)} - e^{-.1(8)} = .1572$

5. a.

- Let X denote the time (in minutes) that elapses before the next car arrives.

- X is exponentially distributed with λ = 360/60 = 6 cars per minute.
  $P(X>.5) = e^{-6(.5)} = .0498$.

b.

- If Y counts the number of cars that will arrive in the next half minute, then Y is a Poisson variable with m = (.5)(6) = 3 cars per half a minute.
  P(Y = 0) = e-3(30)/0! = .0498.

6.

Partial PHStat output:

Probability for X >	
X Value	25
Z Value	0.8
P(X>25)	0.2119

(a) $P(X > 25) = PZ > 0.8) = 0.2119$

Partial PHStat output:

Probability for a Range	
From X Value	10
To X Value	20
Z Value for 10	-2.2
Z Value for 20	-0.2
P(X<=10)	0.0139
P(X<=20)	0.4207
P(10<=X<=20)	0.4068

(b) P(10<X<20) = P(-2.2<Z<-0.2) = 0.4068

Partial PHStat output:

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	2.50%
Z Value	-1.959964
X Value	11.20018
Find X and Z Given Cum. Pctage.	
Cumulative Percentage	97.50%
Z Value	1.959964
X Value	30.79982

(c) $P(X_{\text{lower}} < X < X_{\text{upper}}) = 0.95$

$P(Z < -1.96) = 0.0250$ and $P(Z < 1.96) = 0.9750$

$$Z = -1.96 = \frac{X_{\text{lower}} - 21}{5} \qquad Z = +1.96 = \frac{X_{\text{upper}} - 21}{5}$$

$X_{\text{lower}} = 11.2002$ and $X_{\text{upper}} = 30.7998$

7.

(a) PHStat output:

Exponential Probabilities

Data	
Mean	2
X Value	1

Results	
P(<=X)	0.8647

$P(\text{arrival time} \le 1) = 0.8647$

(b) PHStat output:

Exponential Probabilities

Data	
Mean	2
X Value	5

Results	
P(<=X)	0.999955

$P(\text{arrival time} \le 5) = 0.99996$

(c) PHStat output:

Exponential Probabilities

Data	
Mean	1
X Value	1

Results	
P(<=X)	0.6321

Exponential Probabilities

Data	
Mean	1
X Value	5

Results	
P(<=X)	0.993262

If $\lambda = 1$, $P(\text{arrival time} \le 1) = 0.6321$, $P(\text{arrival time} \le 5) = 0.9933$

8. $\lambda = 1/10$

a)

$$P(X > 15) = e^{-15\lambda} = e^{-3/2} = 0.22$$

b)

$$P(X > 15|X > 10) = P(X > 5) = e^{-1/2} = 0.604$$